## Electrical Theory

## Power Flow on AC Transmission Lines

PJM State \& Member Training Dept.

## Objectives

- Describe the basic make-up and theory of an AC transmission line
- Given the formula for real power and information, calculate real power flow on an AC transmission facility
- Given the formula for reactive power and information, calculate reactive power flow on an AC transmission facility
- Given voltage magnitudes and phase angle information between 2 busses, determine how real and reactive power will flow


## Introduction



## AC Power Flow Overview

## AC Power Flow Overview



- Different lines have different values for $R, X_{L}$, and $X_{C}$, depending on:
- Length
- Conductor spacing
- Conductor cross-sectional area
- $X_{C}$ is equally distributed along the line


## Resistance in AC Circuits

- Resistance $(R)$ is the property of a material that opposes current flow causing real power or watt losses due to $I^{2} R$ heating
- Line resistance is dependent on:
- Conductor material
- Conductor cross-sectional area
- Conductor length
- In a purely resistive circuit, current and voltage are in phase; instantaneous power equaling the product of the two


## Power in Out-of-Phase AC Circuits

- Reactance is the opposition to current caused by capacitors and inductors
- Reactance causes current to be out-of-phase with voltage
- Inductive reactance $\left(\mathrm{X}_{\mathrm{L}}\right)$ causes current to lag the voltage
- Capacitive reactance ( $\mathrm{X}_{\mathrm{C}}$ ) causes current to lead the voltage
- Loads containing pure inductance or pure capacitance cause the current to be exactly $90^{\circ}$ out of phase with the voltage



## Power in Out-of-Phase AC Circuits

- Inductance and capacitance depend on:
- Conductor cross-sectional area
- Conductor length
- Distance between phase conductors
- Inductive reactance:
- Decreases as the cross-sectional area of the conductor increases
- Increases as the conductor spacing increases
- Capacitive reactance:
- Increases as the cross-sectional area of the conductor increases
- Decreases as the conductor spacing increases


## Power in Out-of-Phase AC Circuits

- Capacitance is always greater for underground cables where conductors and ground are very close
- AC voltage causes the charge on the conductors to increase and decrease as the voltage increases and decreases
- Charging Current is current that flows due to the alternate charge and discharge of the line due to alternating voltage regardless of whether the circuit is closed or open-ended


## Review

- Total impedance $(Z)$ is made up of resistance, inductance, and capacitance
- The reactive component $(X)$ of impedance is made up of inductance and capacitance and is greater than the resistive ( $R$ ) component of a line
- The Reactive components magnitude correlate with the voltage level


## Flow of Real Power

## Power Flow

- MW flow on a transmission facility is the result of the resistive component ( R )
- Real power is measured in watts (W) and is in-phase with the load
- VAR flow on a transmission facility is the result of the reactive component (X)
- Reactive power is measured in volt amperes reactive (VAR) and is out of phase with the load
- VARs supply magnetizing current for inductive loads and charging current for capacitive loads


## Real Power Flow

Real Power $\left(\mathrm{P}_{\mathrm{R}}\right)$ flow between two buses is obtained by:

Where,

$$
P_{R}=\frac{V_{S} \times V_{R}}{X} \times \sin \delta
$$

$$
\begin{aligned}
& \mathrm{P}=\text { Real power in MW } \\
& \mathrm{V}_{\mathrm{S}}=\text { Sending-end voltage } \\
& \mathrm{V}_{\mathrm{R}}=\text { Receiving-end voltage } \\
& \mathrm{X}=\text { Line impedance between buses } \\
& \delta=\text { Angle delta between bus voltages }
\end{aligned}
$$

## Real Power Flow

- Angle theta, $\theta$ is the symbol for the angle difference between current and voltage
- Used in determining power factor indicating the portion of total current and voltage that is producing real power
- Angle delta, $\delta$ is the symbol for phase angle difference between the sending and receiving voltages
- Negative MW's indicate flow into the sending bus Positive MW's indicate flow out of the sending bus


## Real Power Flow




Angular difference between buses

## Real Power Flow

- In order to transfer real power across a transmission line, there must be an angle (delta) between the voltages at each end of the line
- Greater phase angle difference; more real power transferred
- Maximum power transfer theoretically occurs at $90^{\circ}$
- Real Power flows "downhill" to a more lagging angle


## Flow of Real Power Summary

- Increasing impedance results in a decrease in real power transfer
- Increasing the phase angle difference increases real power transfer
- Neither increasing or decreasing voltage magnitudes has a significant effect on the flow of real power
- If impedances of parallel lines are equal, power flow is equally distributed
- If impedances of parallel lines are different, real power flow is inversely proportional to line impedance


## Flow of Reactive Power

## Flow of Reactive Power

- Reactive Power $\left(P_{Q}\right)$ flow on a transmission line is a result of the inductive reactance of the load requirement and is obtained by:

$$
P_{Q}=\left\{\frac{\left(V_{S}\right)\left(V_{S} \times V_{R}\right)}{X}\right\} \cos \delta
$$

Where,
Q = Reactive Power in MVAR
$\mathrm{V}_{\mathrm{S}}=$ Sending-end voltage
$\Delta V=$ Difference between bus voltages $V_{S}$ and $V_{R}$
$X \quad=$ Line Impedance between buses
$\delta=$ Phase angle between VS and VR

## Flow of Reactive Power

- VARS flow only if there is a difference in bus voltage potential
- VAR's flow downhill from a higher per unit value to a lower per unit value of voltage
- Reactive power flow is similar to real power flow:
- Negative VAR value indicates flow into the reference bus
- Positive VAR value indicates flow out of the reference bus



## Answer Questions 1-4

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## Question \#1

- The voltage at Bus A (reference) is 522 kV at an angle of -15 degrees. The voltage at Bus B is 518 kV at an angle of -18 degrees. Total impedance of the transmission path between the buses is 50 ohms. What is (a) the real power flow, and (b) what is its direction?
(a) $P_{R}=\left\{\frac{\left(V_{A}\right)\left(V_{B}\right)}{X}\right\} \sin \delta=$

$$
\begin{aligned}
& \left\{\frac{(522)(518)}{50}\right\} \sin \left\{-15^{\circ}-\left(-18^{\circ}\right)\right\} \\
= & \left\{\frac{(270,396)}{50}\right\} \sin \left\{3^{\circ}\right\} \\
= & (5,408)(0.0523)=283 \mathrm{MW}
\end{aligned}
$$

(b) Direction of flow is from bus $A$ to bus $B$

## Question \#2

- Calculate the a) real power, and b) it's direction of flow using the following information:

Bus A voltage is 235 kV at an angle of - 23 degrees with a power factor of 0.79
Bus $B$ voltage is 237 kV at an angle of -21 degrees with a power factor of 0.85
Total impedance of the transmission path between the buses is 45 ohms
(a) $P_{R}=\left\{\frac{\left(V_{A}\right)\left(V_{B}\right)}{X}\right\} \sin \delta=\quad\left\{\frac{(235)(237)}{45}\right\} \sin \left\{-23^{\circ}-\left(-21^{\circ}\right)\right\}$

$$
=\left\{\frac{(55,695)}{45}\right\} \sin \left\{-2^{\circ}\right\}
$$

$$
=(1,237.6)(-0.0349)=-43.2 M W
$$

(b) Direction of flow is from Bus $B$ to Bus $A$

## Question \#3

- Bus A has a voltage of 234 kV at an angle of - 18 degrees, and Bus B voltage is 239 kV at an angle of -25 degrees. If the impedance of the transmission path is 45 ohms, what is (a) the reactive power flow between the buses, and (b) in what direction is the reactive power flowing?

$$
\text { (a) } \begin{aligned}
P_{Q}=\left\{\frac{\left(V_{S}\right)(\Delta V)}{X}\right\} \cos \delta= & \left\{\frac{(239)(239-234)}{45}\right\} \cos \left\{-25^{\circ}-\left(-18^{\circ}\right)\right\} \\
= & \left\{\frac{(239)(5)}{45}\right\} \cos \left\{-7^{\circ}\right\} \\
& =(26.55)(0.9925)=26.4 \text { MVAR }
\end{aligned}
$$

(b) Flow is from Bus B to Bus $A$

## Question \#4

- Calculate the a) reactive power between Bus A and Bus B, and b) it's direction of flow using the following information:

Bus A voltage is 547 kV at an angle of -15 degrees with a power factor of 0.78 Bus B voltage is 544 kV at an angle of -18 degrees with a power factor of 0.81 Total impedance of the transmission path is 86 ohms

$$
\text { (a) } \begin{aligned}
P_{Q}=\left\{\frac{\left(V_{S}\right)(\Delta V)}{X}\right\} \cos \delta=\quad & \left\{\frac{(547)(547-544)}{86}\right\} \cos \left\{-15^{\circ}-\left(-18^{\circ}\right)\right\} \\
= & \left\{\frac{(547)(3)}{86}\right\} \cos \left\{3^{\circ}\right\} \\
= & (19.1)(0.998)=19.07 \text { MVAR }
\end{aligned}
$$

(b) Flow is from Bus A to Bus B

## Flow of Reactive Power Summary

- Increasing the voltage magnitude at the sending end increases the reactive power flow toward the receiving end
- Increasing the voltage magnitude at the receiving end decreases the reactive power flow toward the receiving end
- Increasing the path impedance between the two buses decreases the reactive power flow towards the receiving end


# Integrating Real \& Reactive Power Flows 

## Scenario 1

- Voltages are in-phase; Bus A voltage > Bus B voltage

- No MW flow; no phase angle difference
- VAR's flow from Bus A to Bus B


## Scenario 2

- Voltages are in-phase; Bus A voltage < Bus B voltage

- No MW flow; no phase angle difference
- VAR's flow from Bus B to Bus A


## Scenario 3

- Voltages are not in-phase; Bus A voltage > Bus B voltage

- MW flow; Bus B voltage is lagging Bus A voltage
- VAR's flow from Bus A to Bus B


## Scenario 4

- Voltages are not in-phase; Bus A voltage > Bus B voltage

- MW flow; Bus A voltage is lagging Bus B voltage
- VAR's flow from Bus A to Bus B


## Scenario 5

- Voltages are not in-phase; Bus A voltage < Bus B voltage

- MW flow; Bus B voltage is lagging Bus A voltage
- VAR's flow from Bus B to Bus A


## Scenario 6

- Voltages are not in-phase; Bus A voltage < Bus B voltage

- MW flow; Bus A voltage is lagging Bus B voltage
- VAR's flow from Bus B to Bus A


## Scenario 7

- Voltages are not in-phase; Bus A voltage = Bus B voltage

- MW flow
- Bus A voltage lags Bus B voltage, MW flow into Bus A
- Bus B voltage lags Bus A voltage, MW flow out of Bus A
- VAR's flow from Bus B and from Bus A into the line


## Answer Question 5

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## Question \#5

- Bus A has a voltage of 527 kV at an angle of -16 degrees. Bus $B$ voltage is equal to 542 kV at an angle of -19 degrees. Load 1 has a power factor of 0.93 , while load 2 has a power factor of 0.82 . If the total impedance of the transmission path is 75 ohms, find (a) the real power flow, (b) the direction of real power flow, (c) the reactive power flow, and (d) the direction of reactive power flow?



## Question \#5

$$
\begin{aligned}
P_{R} & =\left(V_{S}\right)\left(V_{R}\right) / Z * \sin ^{\delta} \\
& =(527 \mathrm{kV})(542 \mathrm{kV}) / 75 \Omega * \sin (-16-(-19)) \\
& =3808.4 * 0.052 \\
& =199.3 \mathrm{MW}
\end{aligned}
$$

Real Power will flow from Bus A to Bus B

$$
\begin{aligned}
P_{\mathrm{Q}} & =\left(\mathrm{V}_{\mathrm{S}}\right)\left(\mathrm{V}_{\mathrm{S}}-\mathrm{V}_{\mathrm{R}}\right) / \mathrm{Z} * \cos ^{\delta} \\
& =(542 \mathrm{kV})(542 \mathrm{kV}-527 \mathrm{kV}) / 75 \Omega * \cos (-19-(-16)) \\
& =108.4 * 0.998 \\
& =108.2 \text { MVAR }
\end{aligned}
$$

Reactive Power will flow from Bus $B$ to Bus $A$

## Summary

- Reviewed the three basic components that make up a transmission line: resistance, inductance, and capacitance
- Applied the formula for calculating real power flow on an AC transmission line
- Applied the formula for calculating reactive power flow on an AC transmission line
- Given voltage and phase angle information, interpreted how real and reactive power will flow on a transmission facility


## Questions?

# PJM Client Management \& Services <br> Telephone: (610) 666-8980 <br> Toll Free Telephone: (866) 400-8980 <br> Website: www.pim.com 

## Member? <br> $\cdots$ Community

The Member Community is PJM's self-service portal for members to search for answers to their questions or to track and/or open cases with Client Management \& Services

## Resources and References

- Rustebakke, Homer M. (1983). Electric Utility Systems and Practices. New York: John Wiley and Sons
- Miller, Robert H., \& Malinowski, James H. (1993). Power System Operation. New York: McGraw-Hill Inc.

