Regulation Market Optimization

RMISTF

June 22, 2016

Howard Haas

Monitoring Analytics

Basics of approach: Isoquant

- **Isoquant:**
- **Set of points that defines combinations of inputs that provide a fixed output. Shows that the output is a defined function of the two inputs.**

- **Regulation Isoquant:**
- **Set of combinations of RegD MW and RegA MW that provide an expected level of ACE control.**

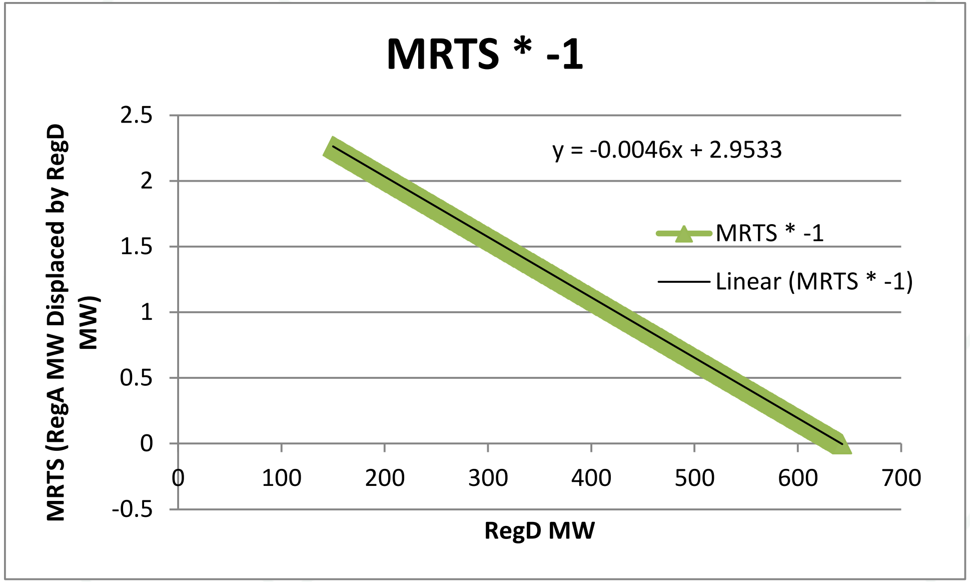
PJM Modeled Control Scores for various new signal based RegA/RegD combinations

Average of Con	RegA																				
RegD	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000
0									260.4676	253.52	248.3677	244.6573	242.2628	240.7543	239.9615	239.7899	240.2729	241.2953	242.7395	244.6651	246.8473
50								238.2629	229.0765	222.6455	218.013	214.7082	212.4433	211.1378	210.4281	210.2994	210.8478	211.7306	212.9072	214.3292	
100							228.2232	215.4021	205.4934	198.2	192.9836	188.9761	185.8499	184.129	183.0455	182.5161	182.5368	182.6374	182.8033		
150						228.0722	209.5654	195.4245	184.7992	176.5644	170.5145	165.8044	161.941	159.2971	157.4591	156.1529	155.3979	155.1371			
200					233.9134	212.0164	193.1807	178.1102	166.6785	157.9115	151.3098	145.6437	141.1636	138.1651	135.5467	133.5958	132.2064				
250				245.6584	219.5599	197.3763	178.6492	163.3712	151.3158	142.426	135.1577	129.2936	124.6338	121.2239	118.0783	115.8994					
300			263.4692	234.0175	207.258	184.5939	165.7654	150.6714	138.4881	129.8193	122.4056	116.7235	112.1725	108.7225	105.608						
350		286.7793	253.7045	223.8123	196.4878	173.5508	154.8205	140.0527	128.2034	119.6716	112.7312	107.2895	103.2071	99.75885							
400	314.2612	278.4291	245.054	214.5861	186.9844	164.0033	145.4262	131.4191	120.2336	112.18	105.4005	100.4343	96.46236								
450	306.7571	271.0234	236.9429	206.1838	178.5909	155.6877	137.8448	124.4198	114.2869	106.3168	100.141	95.40625									
500	300.1569	264.2888	229.7802	198.5891	171.247	148.3857	131.3197	118.7627	109.2292	101.869	96.14533										
550	294.1045	258.0281	223.5786	191.8847	164.5043	142.1707	126.0324	113.9991	105.1491	98.19274											
600	288.4192	252.372	217.5941	186.0489	158.6807	137.0086	121.5723	110.2575	101.8961												
650	282.9796	247.1173	212.1962	180.5825	153.5373	132.6016	117.9852	107.0773													
700	277.8865	242.4719	207.3627	175.6695	148.7507	128.9552	114.7422														
750	273.279	237.9201	202.9188	171.631	144.5769	125.6129															
800	268.7797	233.5976	198.6674	167.6297	141.1609																
850	264.3141	229.5828	194.8414	163.839																	
900	260.223	225.735	191.2823																		
950	256.4235	222.0159																			
1000	252.7491																				

Basics of approach: MRTS

- **MRTS = Marginal Rate of Technical Substitution.**
- **The slope of the isoquant at any point (where a point is a combination of inputs) for a specific level of fixed output. Defines the marginal rate of substitution between inputs at each point.**
- **The rate of substitution between inputs holding output constant.**
- **An exchange rate that converts substitutable inputs into common units so that they can be compared directly in optimization and in the market.**

PJM based combinations: MRTS



MRTS = Point specific slopes of the isoquant defining the rate of substitution.

Derivative of curve defining combinations of RegA/RegD

Basics of approach: MRTS

- MRTS: The rate of substitution between RegD and RegA
- Example:

  MRTS = (MRTS of D MW for A MW) = 2.
- Indicates that at this point on the isoquant:
  - 1 D MW can be substituted (1 MW D x MRTS = 2) for 2 MW of A at that point on the isoquant.

    OR
  - 2 MW of A can be substituted (2 MW D/MRTS = 1) for 1 MW of D at that point on the isoquant.

Basics of approach: MRTS as exchange rate

- **Using MRTS a RegD offer can be compared directly to a RegA offer.**
  - **If MRTS = (MRTS of D MW for A MW) = 2.**
  - **($20/MW D) / MRTS = offer in terms of $/MW A**
  - **($20 per RegD MW) /2 = $10/MW in terms of equivalent A MW.**
- **Defines whether it is economic to exchange 1 MW of D for MRTS * MW of A or (A MW)/MRTS for 1 MW of D .**
- **Basis of the decision at any point is based on the marginal relative values in terms of output and price at that point.**

Consistent Application of MRTS

- Single clearing price (input) model.
- <u>Resources evaluated and paid on per marginal effective MW basis.</u>
- MRTS converts offers into equivalent units
  - MRTS of A = 1, MRTS of D = MRTS (MW D)
- P = <u>marginal price</u> of Effective MW, highest cost cleared resource (A or D), in terms of $/RegA equivalent.
  - P = Max(MAX(PD (MW D) /MRTS), MAX(PA(MW A))
- Payment is per marginal RegA equivalent MW.
  - Payment = P x MRTS x MW

Example of Market Optimization

June 22, 2016

Howard Haas

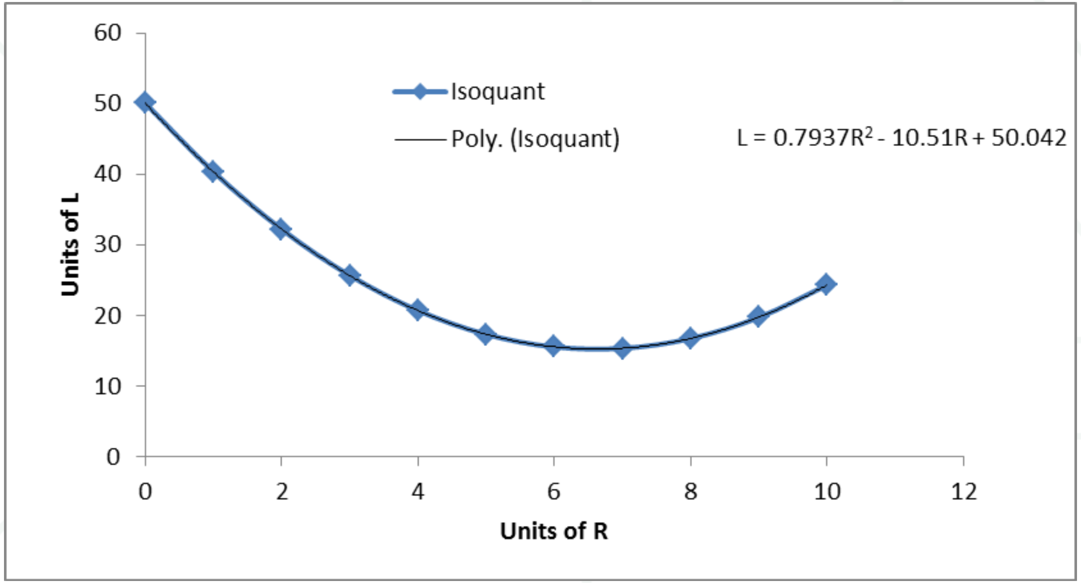
Monitoring Analytics

Basics of approach: Two input production model

Effective Labor	L	R	MRTS	MRTS * -1	Area under Curve Calculation of effective L from K (Displaced L from K)	Residual L	Total Effective L
50.042	50.042	0	-10.51	10.51	0	50.042	50.042
50.042	40.3257	1	-8.9226	8.9226	9.7163	40.3257	50.042
50.042	32.1968	2	-7.3352	7.3352	17.8452	32.1968	50.042
50.042	25.6553	3	-5.7478	5.7478	24.3867	25.6553	50.042
50.042	20.7012	4	-4.1604	4.1604	29.3408	20.7012	50.042
50.042	17.3345	5	-2.57	2.57	32.7075	17.3345	50.042
50.042	15.5552	6	-0.9856	0.9856	34.4868	15.5552	50.042
50.042	15.3633	7	0.6018	-0.6018	34.6787	15.3633	50.042
50.042	16.7588	8	2.1892	-2.1892	33.2832	16.7588	50.042
50.042	19.7417	9	3.7766	-3.7766	30.3003	19.7417	50.042
50.042	24.312	10	5.364	-5.364	25.73	24.312	50.042

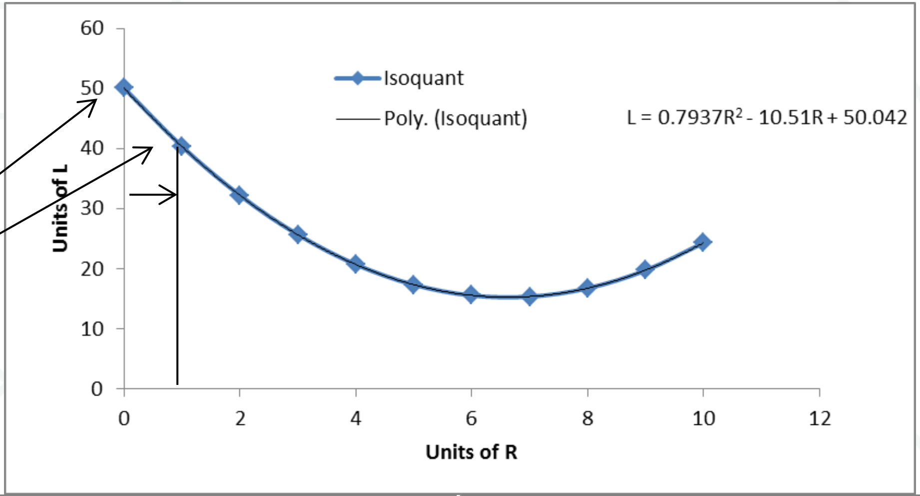
Basics of approach: Isoquant

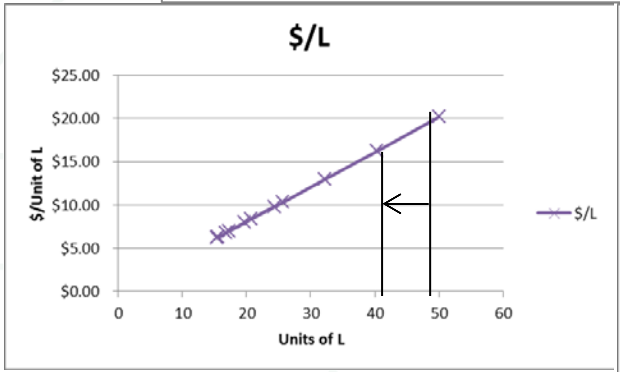
L	R
50.042	0
40.3257	1
32.1968	2
25.6553	3
20.7012	4
17.3345	5
15.5552	6
15.3633	7
16.7588	8
19.7417	9
24.312	10

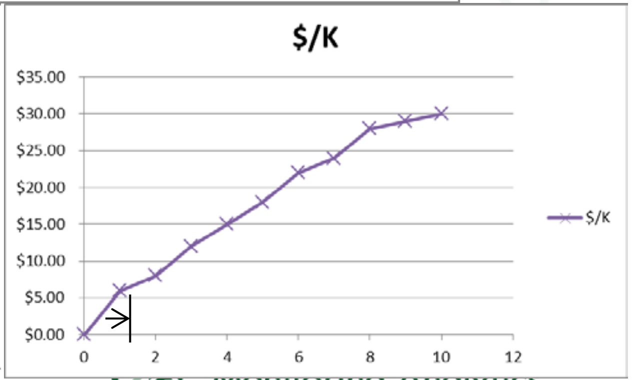


Solve for least cost combination

Labor (L)	Robots (R)	$/L	$/R
50.04	0	$20.18	$0.00
40.33	1	$16.26	$6.00
32.20	2	$12.98	$8.00
25.66	3	$10.34	$12.00
20.70	4	$8.35	$15.00
17.33	5	$7.00	$18.00
15.56	**6**	$6.27	$22.00
15.36	7	$6.19	$24.00
16.76	8	$6.76	$28.00
19.74	9	$7.96	$29.00
24.31	10	$9.80	$30.00

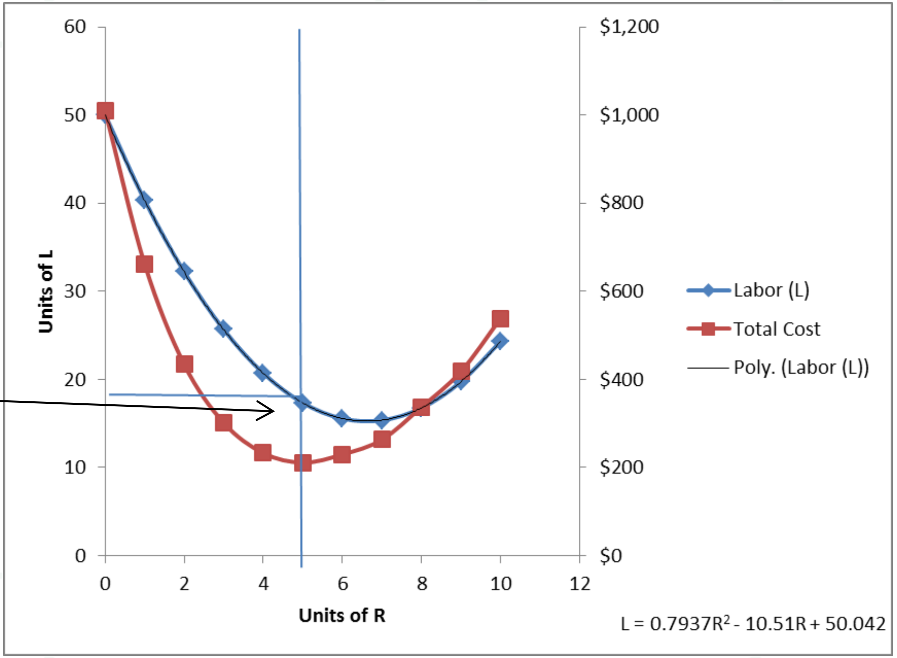






Basics of approach

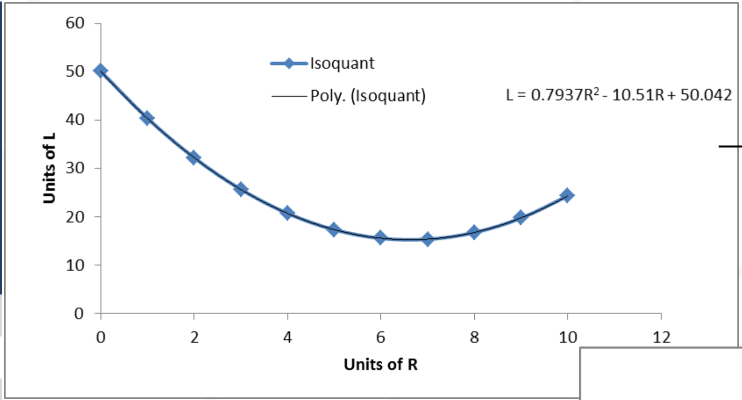
Isoquant				
Labor (L)	Robots (R)	$/L	$/R	Total Cost
50.04	0	$20.18	$0.00	$1,010
40.33	1	$16.26	$6.00	$662
32.20	2	$12.98	$8.00	$434
25.66	3	$10.34	$12.00	$301
20.70	4	$8.35	$15.00	$233
17.33	5	$7.00	$18.00	$211
15.56	**6**	$6.27	$22.00	$230
15.36	7	$6.19	$24.00	$263
16.76	8	$6.76	$28.00	$337
19.74	9	$7.96	$29.00	$418
24.31	10	$9.80	$30.00	$538



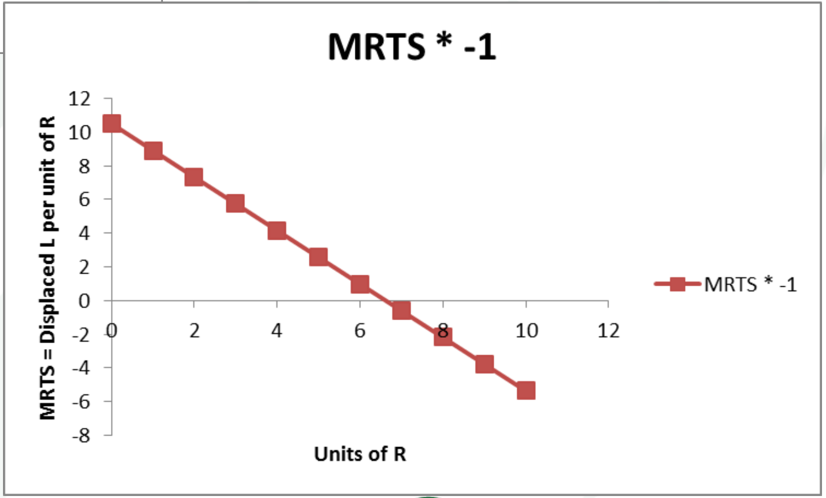
Note, R paid $18/R per R, L paid $7.00 per L.
Will see that at least cost point both R <u>and</u> L are being paid $7.00 per equivalent L.

Derive the MRTS

Isoquant		
Labor (L)	Robots (R)	MRTS
50.04	0	10.51
40.33	1	8.9226
32.20	2	7.3352
25.66	3	5.7478
20.70	4	4.1604
17.33	5	2.57
15.56	**6**	0.9856
15.36	7	-0.6018
16.76	8	-2.1892
19.74	9	-3.7766
24.31	10	-5.364



Point specific slope (derivative) of the Isoquant = MRTS



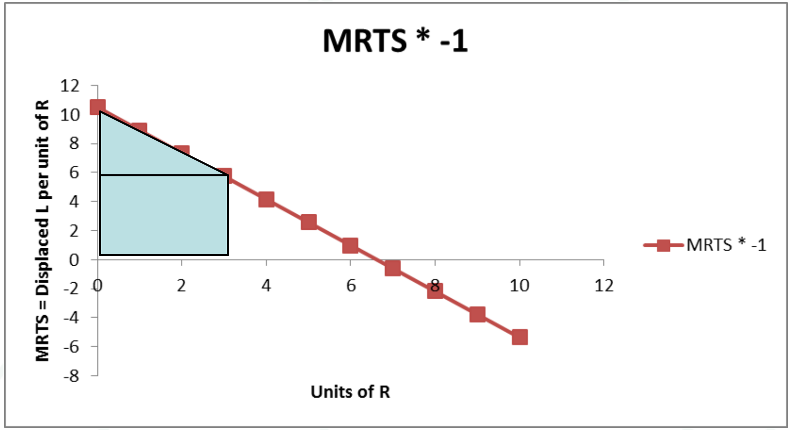
Basics of approach: MRTS as exchange rate

Isoquant		
Labor (L)	Robots (R)	MRTS
50.04	0	10.51
40.33	1	8.9226
32.20	2	7.3352
25.66	3	5.7478
20.70	4	4.1604
17.33	5	2.57
15.56	**6**	0.9856
15.36	7	-0.6018
16.76	8	-2.1892
19.74	9	-3.7766
24.31	10	-5.364

- **MRTS = Change in L/Change in R, holding output constant.**
- **MRTS translates units of R into effective units of L on the margin.**
  - **(1 unit of R * MRTS) = marginal substitution for L**
  - **Total displacement of L by R at any point can be calculated as area under the MRTS curve defined in change in L for change in R.**

Basics of approach: Staying on the curve

Effective Labor	L	R	MRTS * -1	Area under Curve Calculation of effective L from R (Displaced L from R)	Residual L	Total Effective L
50.042	50.042	0	10.51	0	50.042	50.042
50.042	40.3257	1	8.9226	9.7163	40.3257	50.042
50.042	32.1968	2	7.3352	17.8452	32.1968	50.042
50.042	25.6553	3	5.7478	24.3867	25.6553	50.042
50.042	20.7012	4	4.1604	29.3408	20.7012	50.042
50.042	17.3345	5	2.57	32.7075	17.3345	50.042
50.042	15.5552	6	0.9856	34.4868	15.5552	50.042
50.042	15.3633	7	-0.6018	34.6787	15.3633	50.042
50.042	16.7588	8	-2.1892	33.2832	16.7588	50.042
50.042	19.7417	9	-3.7766	30.3003	19.7417	50.042
50.042	24.312	10	-5.364	25.73	24.312	50.042

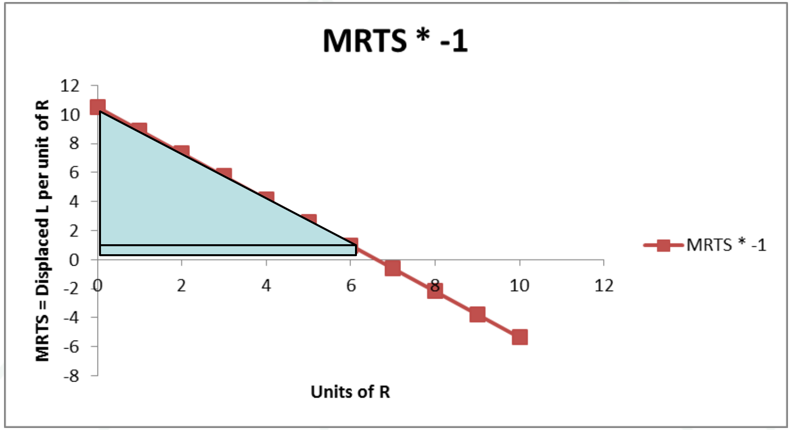


- AT 3 R, Displacing Area in terms of L, holding output constant.
- Producing the equivalent of 50.04 units of L using 3 R and 26.65 L
- 3 K displaces 24.39 L (area under curve).

Monitoring Analytics

Basics of approach: Staying on the curve

Effective Labor	L	R	MRTS * -1	Area under Curve Calculation of effective L from R (Displaced L from R)	Residual L	Total Effective L
50.042	50.042	0	10.51	0	50.042	50.042
50.042	40.3257	1	8.9226	9.7163	40.3257	50.042
50.042	32.1968	2	7.3352	17.8452	32.1968	50.042
50.042	25.6553	3	5.7478	24.3867	25.6553	50.042
50.042	20.7012	4	4.1604	29.3408	20.7012	50.042
50.042	17.3345	5	2.57	32.7075	17.3345	50.042
50.042	15.5552	6	0.9856	34.4868	15.5552	50.042
50.042	15.3633	7	-0.6018	34.6787	15.3633	50.042
50.042	16.7588	8	-2.1892	33.2832	16.7588	50.042
50.042	19.7417	9	-3.7766	30.3003	19.7417	50.042
50.042	24.312	10	-5.364	25.73	24.312	50.042

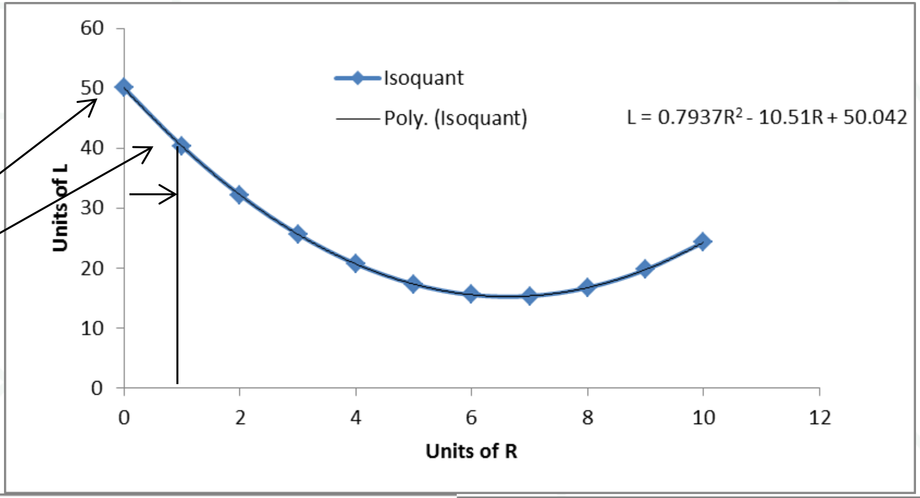


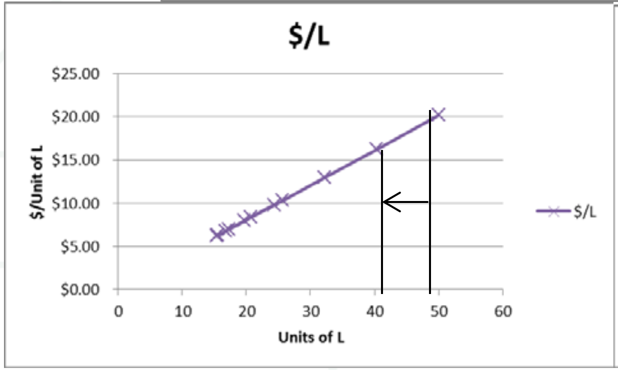
- AT 6 R, Displacing Area in terms of L, holding output constant
- Producing the equivalent of 50.042 units of L using 6 R and 15.5552 L
- 6 R displaces 34.4868 L (area under curve).

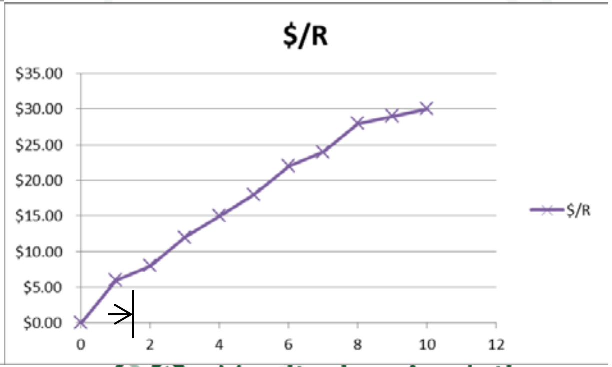
Monitoring Analytics

Solve for least cost combination

Labor (L)	Robots (R)	$/L	$/R
50.04	0	$20.18	$0.00
40.33	1	$16.26	$6.00
32.20	2	$12.98	$8.00
25.66	3	$10.34	$12.00
20.70	4	$8.35	$15.00
17.33	5	$7.00	$18.00
15.56	**6**	$6.27	$22.00
15.36	7	$6.19	$24.00
16.76	8	$6.76	$28.00
19.74	9	$7.96	$29.00
24.31	10	$9.80	$30.00







Basics of approach: MRTS

Labor (L)	Robots (R)	MRTS
50.04	0	10.51
40.33	1	8.9226
32.20	2	7.3352
25.66	3	5.7478
20.70	4	4.1604
17.33	5	2.57
15.56	**6**	0.9856
15.36	7	-0.6018
16.76	8	-2.1892
19.74	9	-3.7766
24.31	10	-5.364

- MRTS = Change in L/Change in R, holding output constant.
- MRTS translates units of R into effective units of L <u>on the margin</u>.
  - (1 R * MRTS) = marginal substitution for L by R
- Can compare cost on a per unit of L basis by dividing $/R offer by MRTS
  - Should exchange R for L when R is less expensive than L on a <u>marginal per effective unit of output basis</u>.

Basics of approach: Direct Offer Comparison

Isoquant

Labor (L)	Robots (R)	MRTS	Output	$/L	$/R	($/R)/MRTS	Total Cost
50.04	0	10.51	10	$20.18	$0.00		$1,010
40.33	1	8.9226	10	$16.26	$6.00	$ 0.67	$662
32.20	2	7.3352	10	$12.98	$8.00	$ 1.09	$434
25.66	3	5.7478	10	$10.34	$12.00	$ 2.09	$301
20.70	4	4.1604	10	$8.35	$15.00	$ 3.61	$233
17.33	5	2.57	10	$7.00	$18.00	$ 7.00	$211
15.56	**6**	0.9856	10	$6.27	$22.00	$ 22.32	$230
15.36	7	-0.6018	10	$6.19	$24.00	$ (39.88)	$263
16.76	8	-2.1892	10	$6.76	$28.00	$ (12.79)	$337
19.74	9	-3.7766	10	$7.96	$29.00	$ (7.68)	$418
24.31	10	-5.364	10	$9.80	$30.00	$ (5.59)	$538

- **To compare cost of R on a per unit of L basis at any point, divide R offer by MRTS.**
- **Should exchange R for L when R is less expensive than L on a marginal per effective unit of output basis.**

Basics of approach: MRTS as exchange rate

Isoquant

Labor (L)	Robots (R)	MRTS	Output	$/L	$/R	($/R)/MRTS	Total Cost
50.04	0	10.51	10	$20.18	$0.00		$1,010
40.33	1	8.9226	10	$16.26	$6.00	$ 0.67	$662
32.20	2	7.3352	10	$12.98	$8.00	$ 1.09	$434
25.66	3	5.7478	10	$10.34	$12.00	$ 2.09	$301
20.70	4	4.1604	10	$8.35	$15.00	$ 3.61	$233
17.33	5	2.57	10	$7.00	$18.00	$ 7.00	$211
15.56	**6**	0.9856	10	$6.27	$22.00	$ 22.32	$230
15.36	7	-0.6018	10	$6.19	$24.00	$ (39.88)	$263
16.76	8	-2.1892	10	$6.76	$28.00	$ (12.79)	$337
19.74	9	-3.7766	10	$7.96	$29.00	$ (7.68)	$418
24.31	10	-5.364	10	$9.80	$30.00	$ (5.59)	$538

$L>$R/MRTS

$L=$R/MRTS

$L<$R/MRTS

- **K and L paid the same in terms of a common unit (either equivalent K or L) per unit at the margin = least cost solution**

Basics of approach: MRTS as exchange rate

Isoquant

Labor (L)	Robots (R)	MRTS	Output	$/L	$/R	($/R)/MRTS	Total Cost
50.04	0	10.51	10	$20.18	$0.00		$1,010
40.33	1	8.9226	10	$16.26	$6.00	$ 0.67	$662
32.20	2	7.3352	10	$12.98	$8.00	$ 1.09	$434
25.66	3	5.7478	10	$10.34	$12.00	$ 2.09	$301
20.70	4	4.1604	10	$8.35	$15.00	$ 3.61	$233
17.33	5	2.57	10	$7.00	$18.00	$ 7.00	$211
15.56	**6**	0.9856	10	$6.27	$22.00	$ 22.32	$230
15.36	7	-0.6018	10	$6.19	$24.00	$ (39.88)	$263
16.76	8	-2.1892	10	$6.76	$28.00	$ (12.79)	$337
19.74	9	-3.7766	10	$7.96	$29.00	$ (7.68)	$418
24.31	10	-5.364	10	$9.80	$30.00	$ (5.59)	$538

- **Unit of L costs > Cost of equivalent unit of L from a unit of R**
  - **($16.26/L from L vs. $0.67/L from R)**
- **Unit of L costs = Cost equivalent unit of L from a unit of R**
  - **($7/L from L vs. $7/L from R)**
- **Unit of L costs < Cost equivalent unit of L from a unit of R**

Basics of approach: MRTS as exchange rate

Isoquant

Labor (L)	Robots (R)	MRTS	Output	$/L	$/R	($/R)/MRTS	Total Cost
50.04	0	10.51	10	$20.18	$0.00		$1,010
40.33	1	8.9226	10	$16.26	$6.00	$ 0.67	$662
32.20	2	7.3352	10	$12.98	$8.00	$ 1.09	$434
25.66	3	5.7478	10	$10.34	$12.00	$ 2.09	$301
20.70	4	4.1604	10	$8.35	$15.00	$ 3.61	$233
17.33	5	2.57	10	$7.00	$18.00	$ 7.00	$211
15.56	**6**	0.9856	10	$6.27	$22.00	$ 22.32	$230
15.36	7	-0.6018	10	$6.19	$24.00	$ (39.88)	$263
16.76	8	-2.1892	10	$6.76	$28.00	$ (12.79)	$337
19.74	9	-3.7766	10	$7.96	$29.00	$ (7.68)	$418
24.31	10	-5.364	10	$9.80	$30.00	$ (5.59)	$538

$L>$R/MRTS

$L=$R/MRTS

$L<$R/MRTS

- **Note, R paid $18/R per R, the same as R being paid $7.00 per equivalent L (MRTS x R x $L)**

Basics of approach: MRTS = Ratio of (unmodified) Prices Paid

Isoquant						
Labor (L)	Robots (R)	$/L	$/R	Total Cost	Slope of Isocost (Ratio of Prices)	MRTS
50.04	0	$20.18	$0.00	$1,010	-	10.51
40.33	1	$16.26	$6.00	$662	0.37	8.92
32.20	2	$12.98	$8.00	$434	0.62	7.34
25.66	3	$10.34	$12.00	$301	1.16	5.75
20.70	4	$8.35	$15.00	$233	1.80	4.16
17.33	5	$7.00	$18.00	$211	2.57	2.57
15.56	**6**	$6.27	$22.00	$230	3.51	0.99
15.36	7	$6.19	$24.00	$263	3.87	-0.60
16.76	8	$6.76	$28.00	$337	4.14	-2.19
19.74	9	$7.96	$29.00	$418	3.64	-3.78
24.31	10	$9.80	$30.00	$538	3.06	-5.36

MRTS = ($/R)/($/L)

2.57 = $18/$7

$7/L = ($18/R)/MRTS = $7/L

- MRTS converts R into L.

1R provides 2.57 units of L.

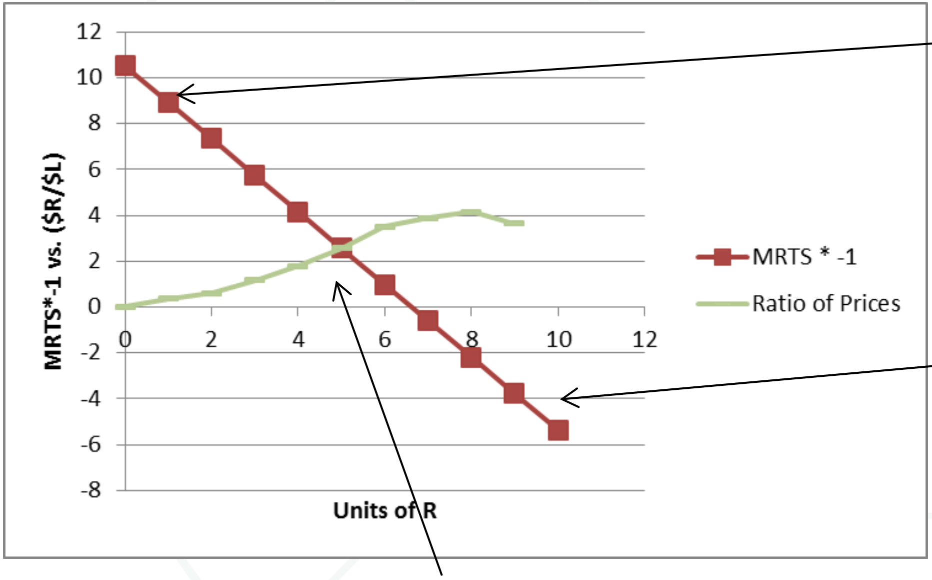
1R costs ($18.00/R)/2.57= $7/L.

1L provides 1 unit of L

1L costs $7/L.

- **R and L paid the same in terms of a common unit (either equivalent R or L) per unit at the margin.**

Basics of approach: MRTS = Ratio of (unmodified) Prices Paid



Where MRTS > $R/$L:

- R is cheaper than L
- 1 R can replace <u>more</u> than 1 L and still produce the same output
- Efficient to replace L with K.

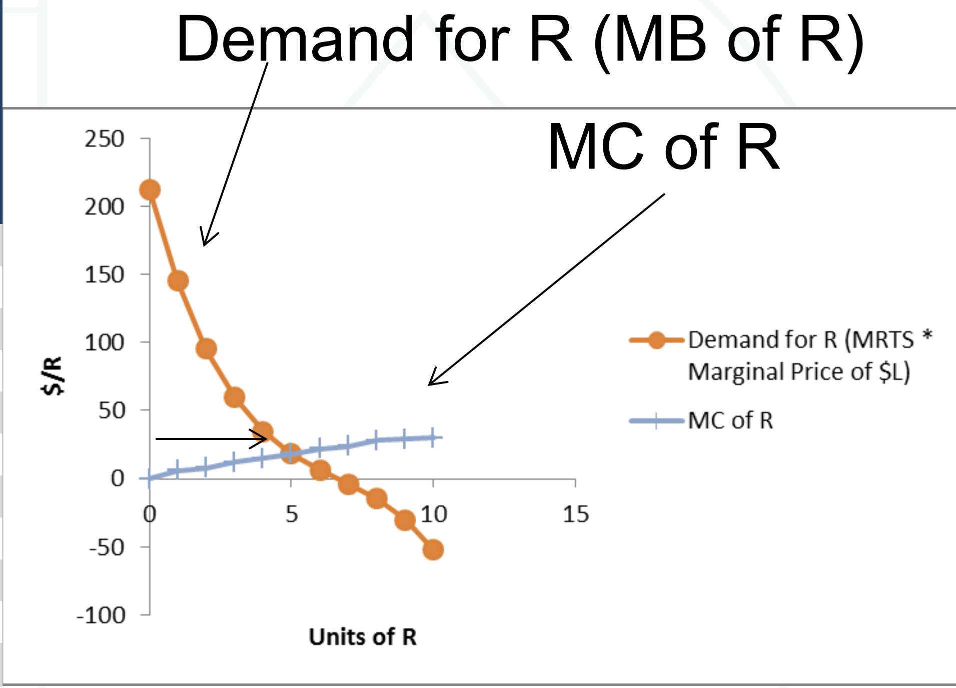
Where MRTS < $R/$L:

- R is more expensive than L.
- 1 R can replace <u>less</u> than 1 L and still produce the same output.
- Efficient to replace R with L.

MRTS = 2.57 = ($/R)/($/L)

Demand for R

Labor (L)	Robots (R)	$/L	$/R	MRTS	Demand for R (MRTS * Marginal Price of $L)	MC of R
Isoquant						
50.04	0	$20.18	$0.00	10.51	$212	$0.00
40.33	1	$16.26	$6.00	8.92	$145	$6.00
32.20	2	$12.98	$8.00	7.34	$95	$8.00
25.66	3	$10.34	$12.00	5.75	$59	$12.00
20.70	4	$8.35	$15.00	4.16	$35	$15.00
17.33	5	$7.00	$18.00	2.57	$18	$18.00
15.56	**6**	$6.27	$22.00	0.99	$6	$22.00
15.36	7	$6.19	$24.00	-0.60	($4)	$24.00
16.76	8	$6.76	$28.00	-2.19	($15)	$28.00
19.74	9	$7.96	$29.00	-3.78	($30)	$29.00
24.31	10	$9.80	$30.00	-5.36	($53)	$30.00



- R and L paid the same in terms of a common unit (either equivalent R or L) per unit at the margin

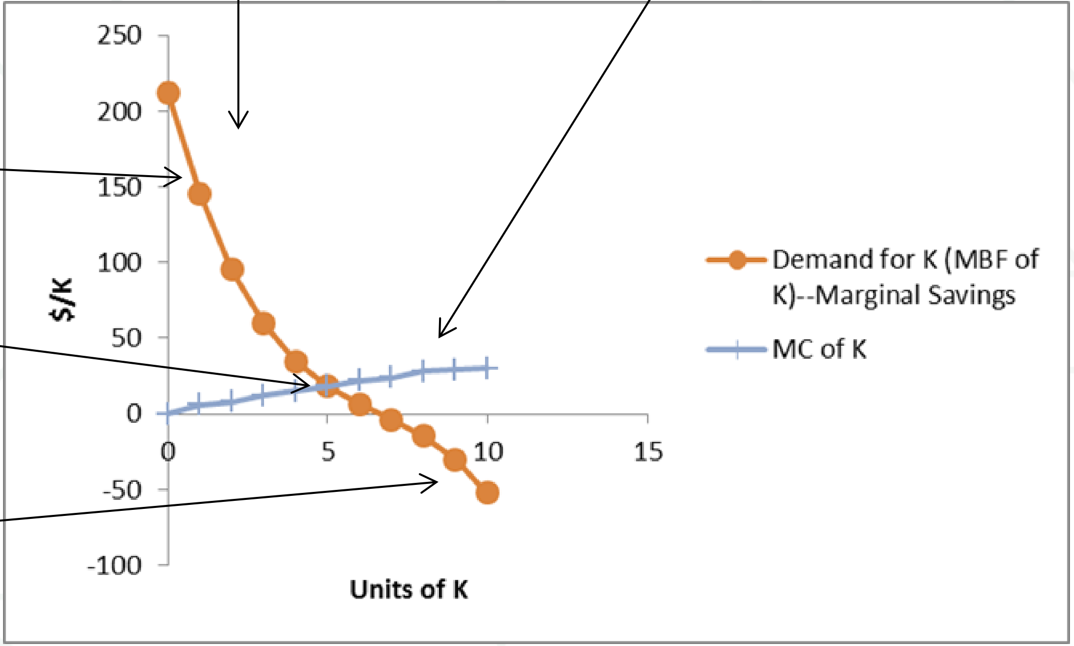
Demand for R

Demand for R (MB of R)

MC of R

$ benefit per unit of R = $ MC of R

Demand for R (MRTS * Marginal Price of $L)	MC of R
$212	$0.00
$145	$6.00
$95	$8.00
$59	$12.00
$35	$15.00
$18	$18.00
$6	$22.00
($4)	$24.00
($15)	$28.00
($30)	$29.00
($53)	$30.00



Demand for R

Isoquant						
Labor (L)	Robots (R)	$/L	$/R	MRTS	Demand for R (MRTS * Marginal Price of $L)	MC of R
50.04	0	$20.18	$0.00	10.51	$212	$0.00
40.33	1	$16.26	$6.00	8.92	$145	$6.00
32.20	2	$12.98	$8.00	7.34	$95	$8.00
25.66	3	$10.34	$12.00	5.75	$59	$12.00
20.70	4	$8.35	$15.00	4.16	$35	$15.00
17.33	5	$7.00	$18.00	2.57	$18	$18.00
15.56	**6**	$6.27	$22.00	0.99	$6	$22.00
15.36	7	$6.19	$24.00	-0.60	($4)	$24.00
16.76	8	$6.76	$28.00	-2.19	($15)	$28.00
19.74	9	$7.96	$29.00	-3.78	($30)	$29.00
24.31	10	$9.80	$30.00	-5.36	($53)	$30.00

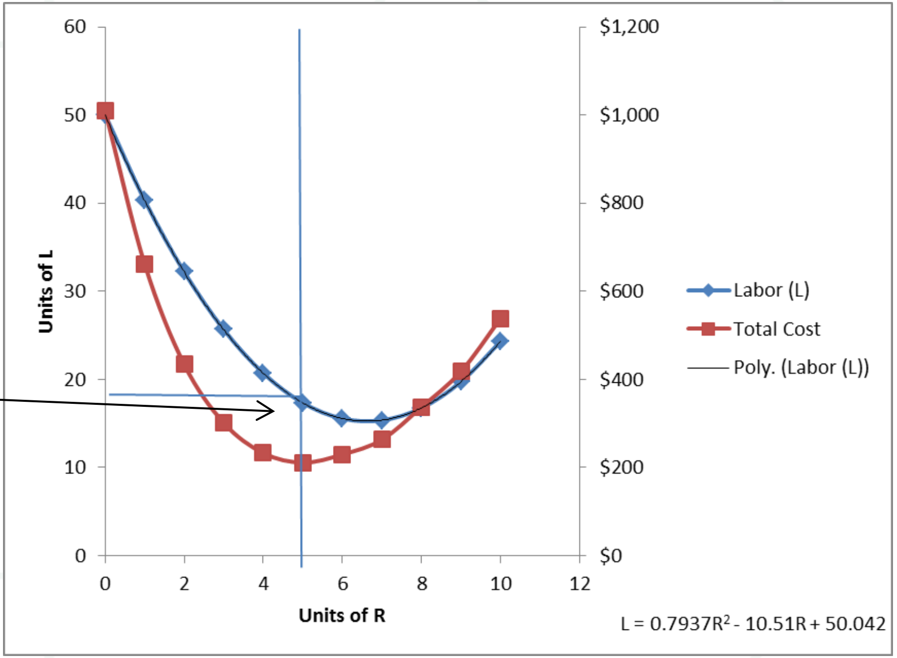
- R and L paid the same in terms of a common unit (either equivalent R or L) per unit at the margin

- **Least cost point:**
  - **R and L paid the same in terms of a common unit (either equivalent R or L) per unit at the margin.**
- **Converted to $/unit of L, clearing price is $7/L.**
- **L is paid $7 per L.**
  - **$7 * L = Payment to L**
- **R is paid $7 per unit of marginal effective L provided.**
  - **$7 * MRTS * R**
  - **$7 * 2.57 * R = $18 * R**

Monitoring Analytics

Basics of approach

Isoquant					
Labor (L)	Robots (R)	$/L	$/R	Total Cost	MRTS
50.04	0	$20.18	$0.00	$1,010	10.51
40.33	1	$16.26	$6.00	$662	8.92
32.20	2	$12.98	$8.00	$434	7.34
25.66	3	$10.34	$12.00	$301	5.75
20.70	4	$8.35	$15.00	$233	4.16
17.33	5	$7.00	$18.00	$211	2.57
15.56	**6**	$6.27	$22.00	$230	0.99
15.36	7	$6.19	$24.00	$263	-0.60
16.76	8	$6.76	$28.00	$337	-2.19
19.74	9	$7.96	$29.00	$418	-3.78
24.31	10	$9.80	$30.00	$538	-5.36



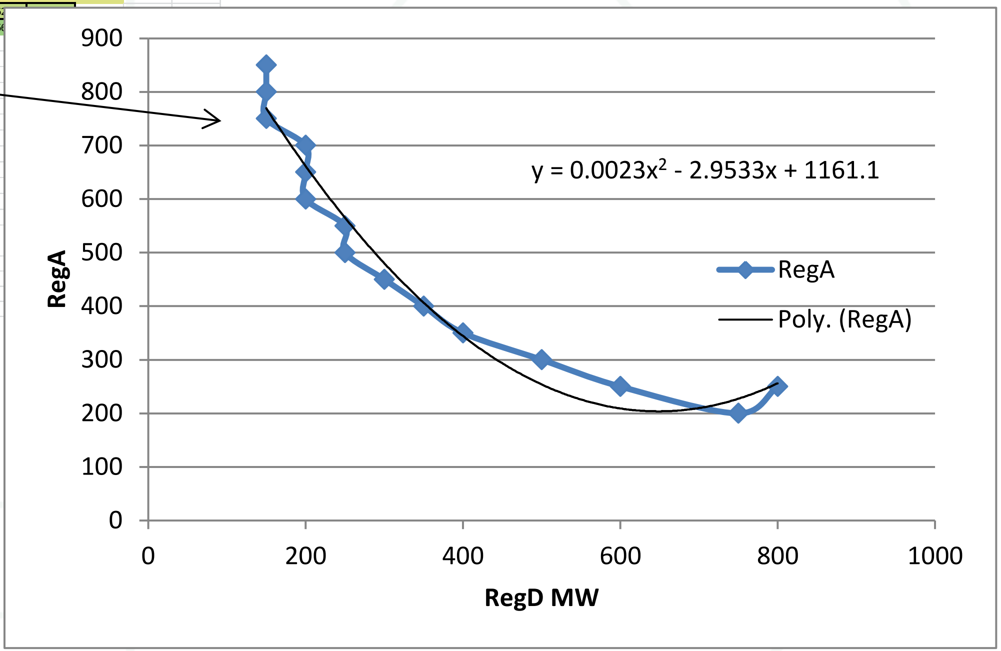
Note, R paid $18/R per R, L paid $7.00 per L.
Will see that at least cost point both R <u>and</u> L are being paid $7.00 per equivalent L.

Direct Application to Regulation Market Design: Need Implementation Consistent with MRTS Definition

Monitoring Analytics

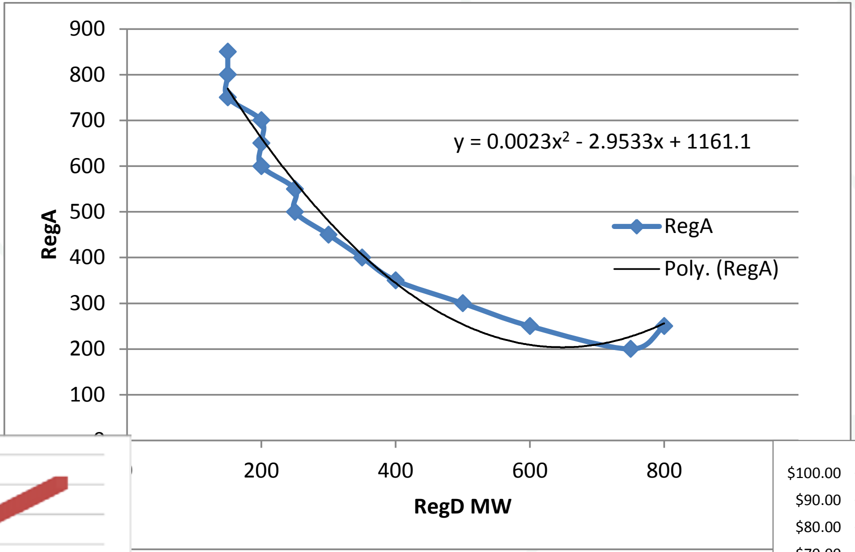
PJM: Assumed Relationship

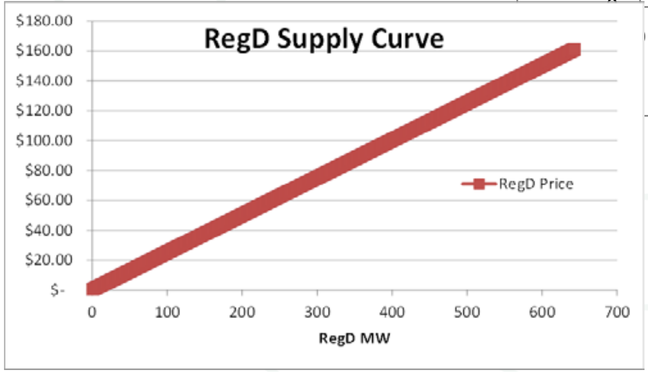
Sum of CPS-1	Column Labels: RegA MW																				
RegD MW	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000
0									143.4	144.275	145.15	145.975	146.8	147.625	148.425	149.2	149.95	150.7	151.45	152.125	152.725
50								145.75	146.75	147.725	148.725	149.775	150.725	151.65	152.6	153.475	154.3	155.075	155.8	156.5	
100							147.7	148.775	150.025	151.175	152.375	153.575	154.725	155.75	156.725	157.675	158.55	159.375	160.15		
150						148.875	150.35	151.75	153.15	154.625	156.025	157.35	158.6	159.775	160.85	161.875	[illegible]				
200					149.35	151.125	152.85	154.5	156.125	157.8	159.375	160.9	162.25	163.5	164.7	165.8	[illegible]				
250				149.175	151.125	153.125	155.05	156.95	158.825	160.7	162.375	163.95	165.45	166.75	168.075	169.1					
300			148.475	150.525	152.725	154.9	156.975	159.075	161.15	163.125	164.9	166.55	168.025	169.475	170.8						
350		147.45	149.575	151.775	154.1	156.45	158.625	160.975	163.2	165.2	167.075	168.75	170.225	171.7							
400	146.425	148.4	150.625	152.925	155.375	157.775	160.125	162.55	164.95	167	168.925	170.55	172.1								
450	147.175	149.275	151.6	154	156.475	159	161.45	163.95	166.325	168.55	170.45	172.1									
500	147.925	150.1	152.5	154.975	157.475	160.05	162.625	165.15	167.6	169.925	171.8										
550	148.6	150.85	153.325	155.875	158.425	161.125	163.725	166.3	168.775	171											
600	149.275	151.55	154.1	156.65	159.325	162.075	164.725	167.325	169.8												
650	149.875	152.2	154.775	157.475	160.15	163	165.6	168.175													
700	150.45	152.775	155.45	158.2	161	163.8	166.4														
750	150.95	153.35	156.025	158.875	161.725	164.525															
800	151.475	153.825	156.625	159.475	162.375																
850	151.975	154.325	157.125	160.025																	
900	152.425	154.825	157.625																		
950	152.85	155.275																			
1000	153.275																				
(blank)																					

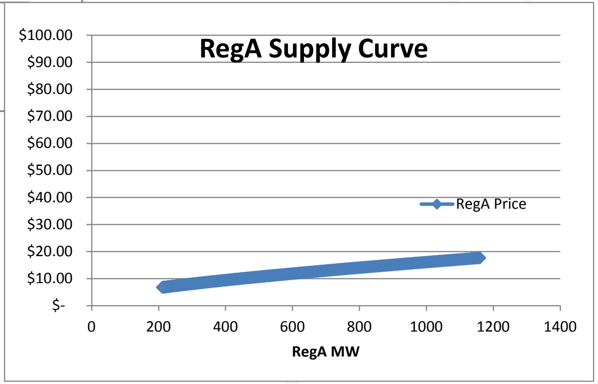


Two inputs, holding output constant.

Solving for least cost combination

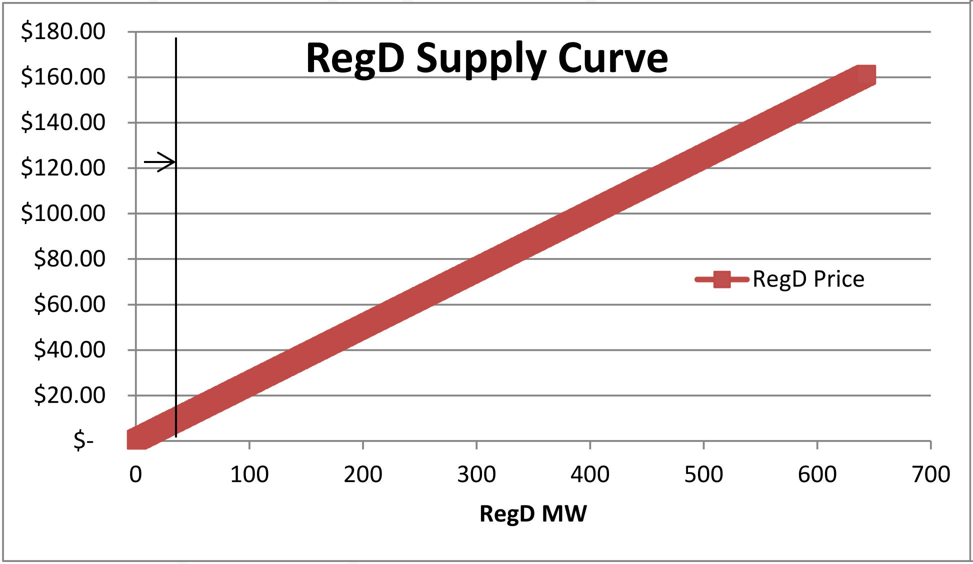


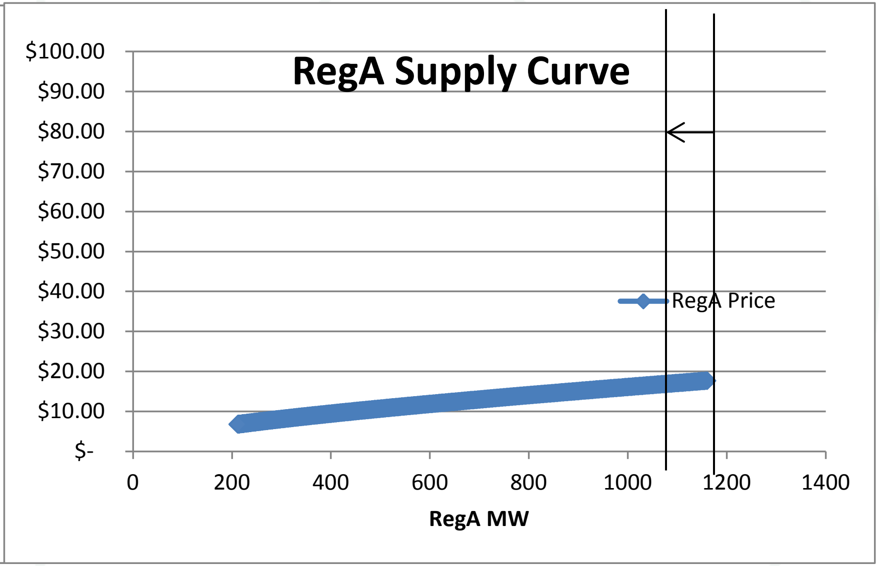




Solving for least cost combination

- Changing the amount of A (or D), changes the amount of D (or A) according to the isoquant.



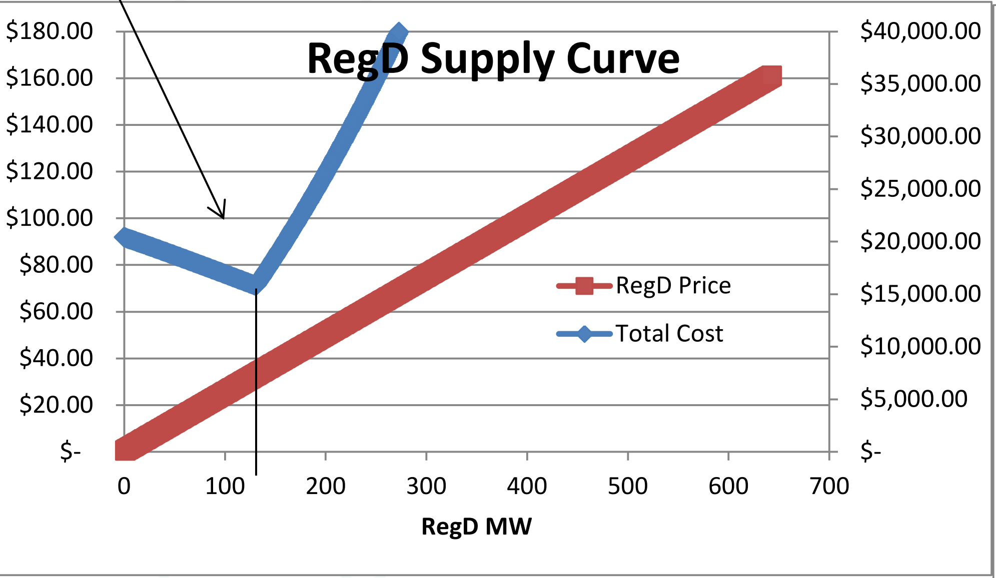


- Rate of change is the slope of the isoquant at point = MRTS.

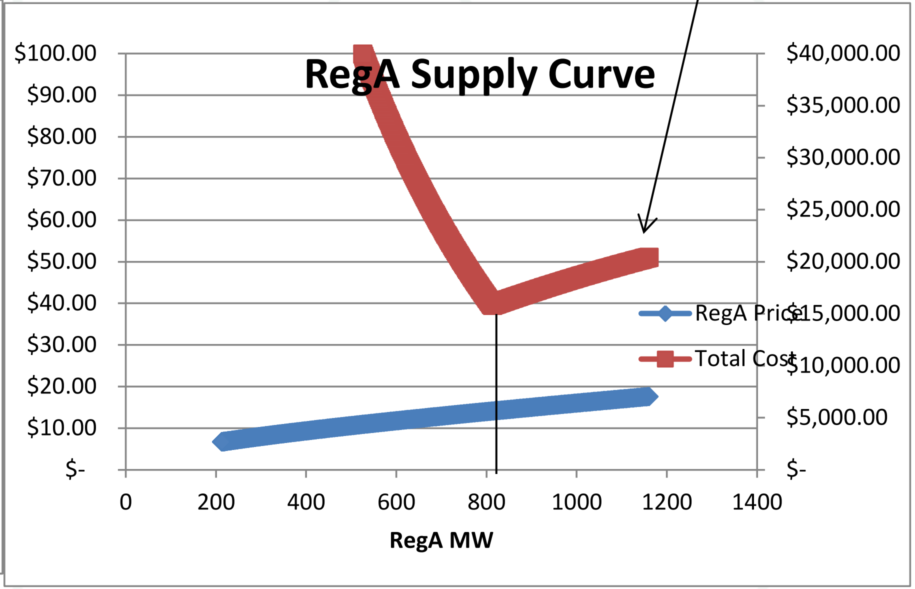
Solving for least cost combination

Find least cost combination

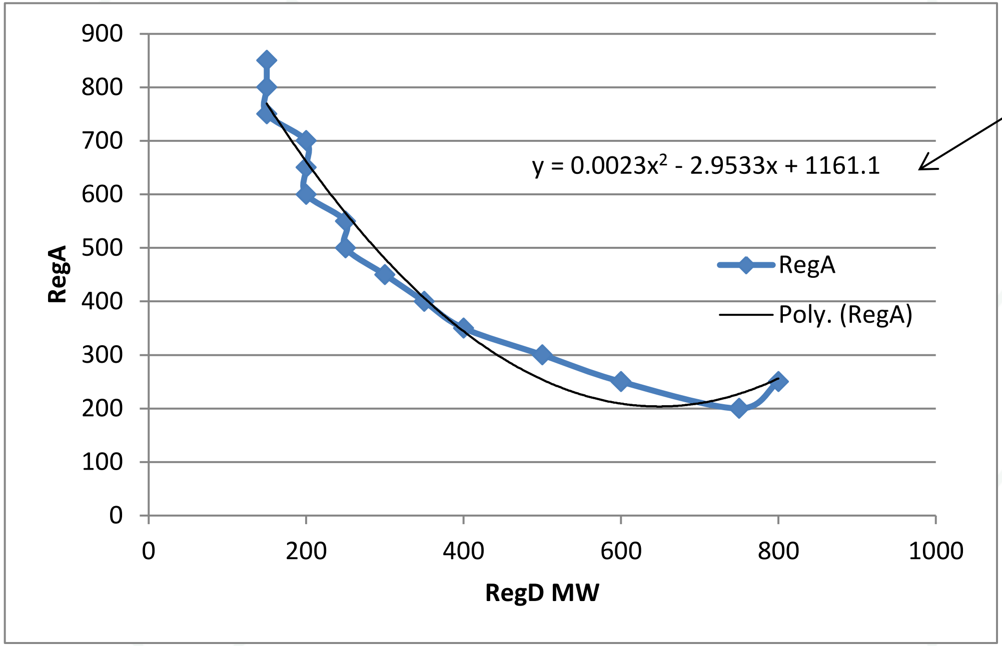
TC



TC



PJM based combinations: Smooth the curve

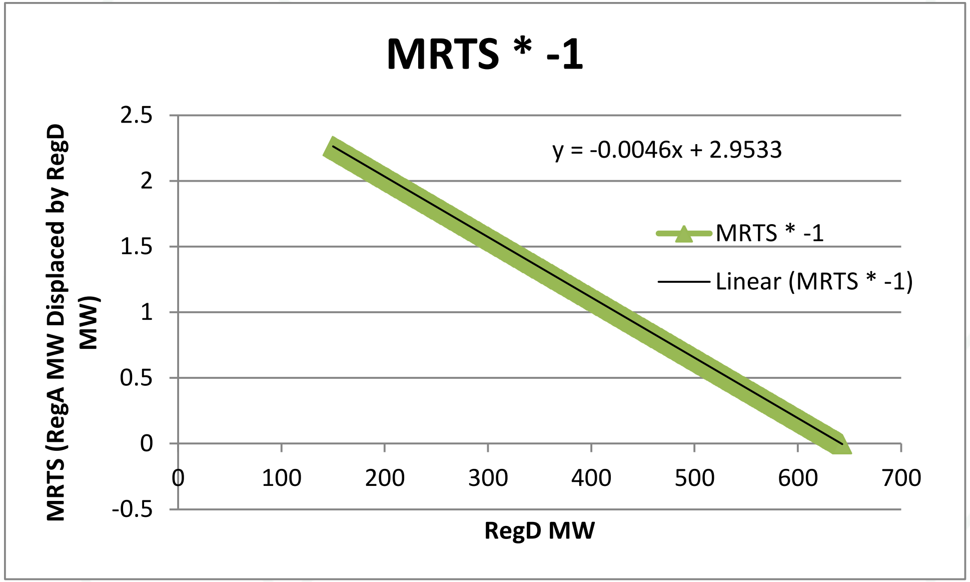


Derivative of this function is MRTS Function

Change in RegA for Change in RegD, holding control metric constant

Monitoring Analytics

PJM based combinations: MRTS



MRTS = Point specific slopes of the isoquant defining the rate of substitution.

Derivative of curve defining combinations of RegA/RegD

Basics of approach: MRTS

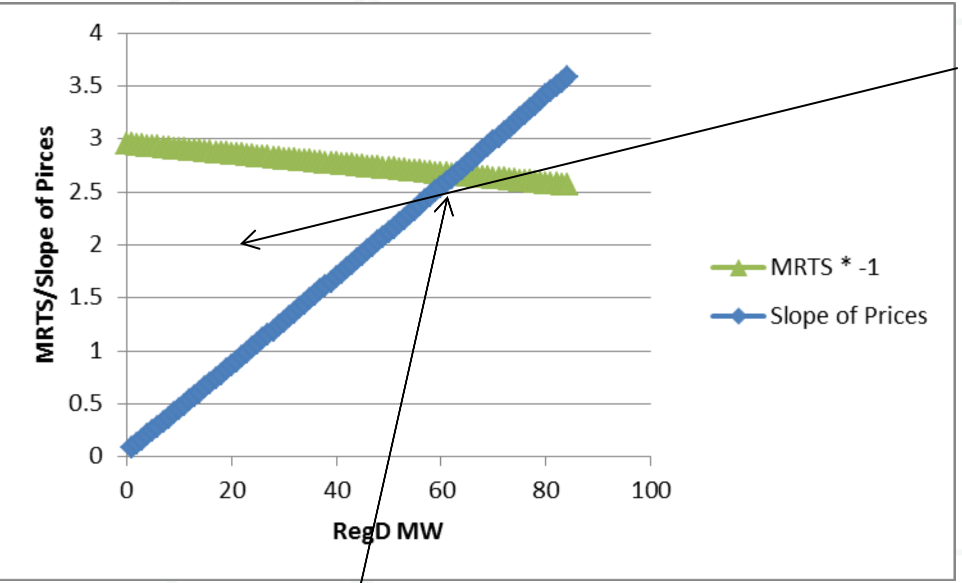
A	D	
~~Labor (L)~~	~~Robots (R)~~	MRTS
50.04	0	10.51
40.33	1	8.9226
32.20	2	7.3352
25.66	3	5.7478
20.70	4	4.1604
17.33	5	2.57
15.56	**6**	0.9856
15.36	7	-0.6018
16.76	8	-2.1892
19.74	9	-3.7766
24.31	10	-5.364

- MRTS = Change in A/Change in D, holding output constant.
- MRTS translates units of D into effective units of A on the margin.
  - (1 D * MRTS) = marginal substitution for A by D
- Can compare cost on a per unit of A basis by dividing $/D offer by MRTS
  - Should exchange D for A when D is less expensive than A on a marginal per effective unit of output basis.

Consistent Application of MRTS

- **Single clearing price (input) model.**
- **Resources evaluated and paid on marginal effective MW basis.**
- **MRTS converts offers into equivalent units**
  - **MRTS of A = 1, MRTS of D = MRTS (MW D)**
- **P = marginal price of Effective MW, highest cost cleared resource (A or D), in terms of $/RegA equivalent.**
  - **P = Max(MAX(PD (MW D) /MRTS), MAX(PA(MW A))**
- **Payment is per marginal RegA equivalent MW.**
  - **Payment = P x MRTS x MW**

Basics of approach: MRTS = Ratio of (unmodified) Input Prices



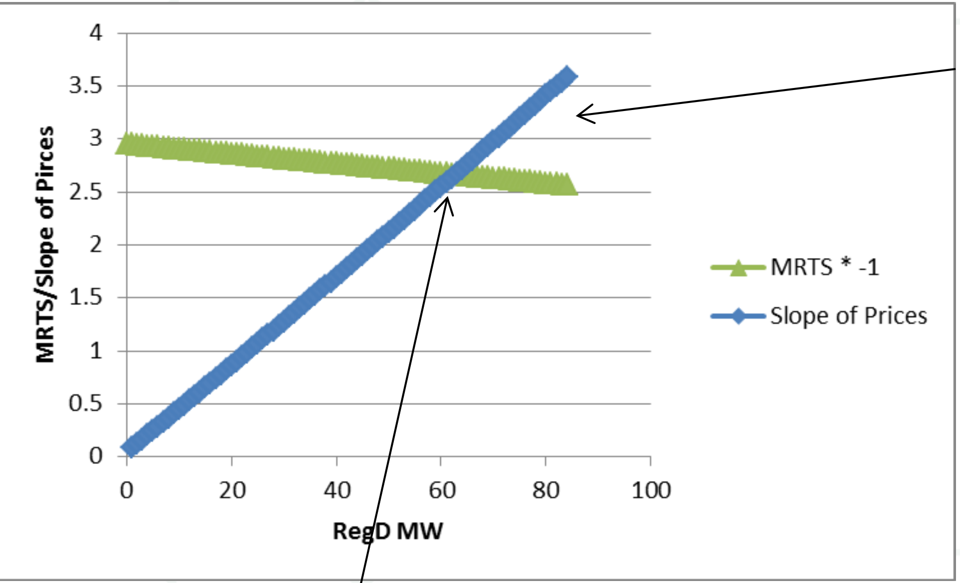
Where MRTS > $D/$A:

- Rate of substitution (D for A) > Ratio of prices (Cost of D relative to A)
- Example:
    - D is less expensive than A.
    - Can use 1 D to replace more than 1 A and still produce the same output.
    - Efficient to replace A with D.

MRTS = ($/D)/($/A)

Monitoring Analytics

Basics of approach: MRTS = Ratio of (unmodified) Input Prices

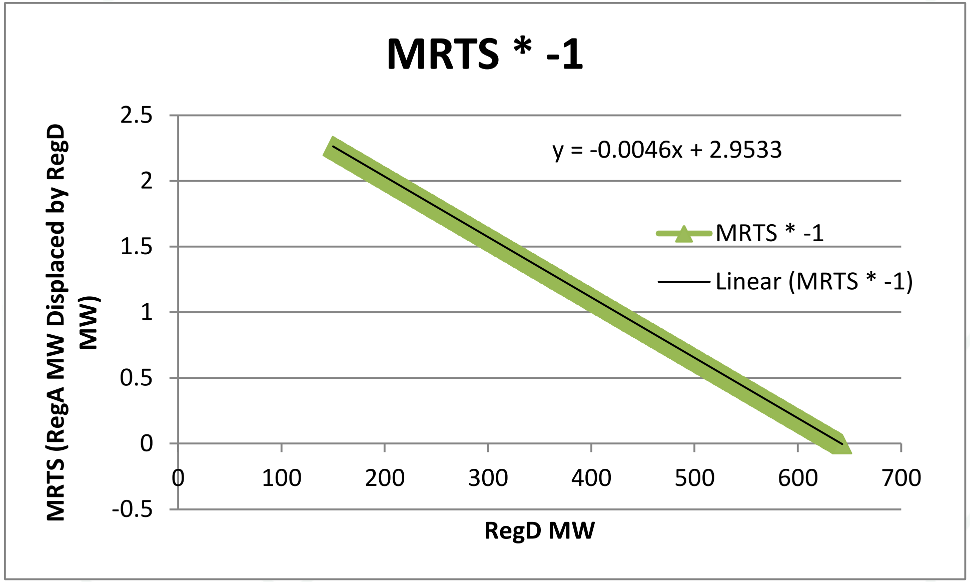


Where MRTS < $D/$A:

- Rate of substitution (D for A) < Ratio of prices (Cost of D relative to A)
- Example:
  - D is more expensive than A.
  - Can use 1 D to replace less than 1 A and still produce the same output.
  - Efficient to replace D with A.

MRTS = ($/D)/($/A)

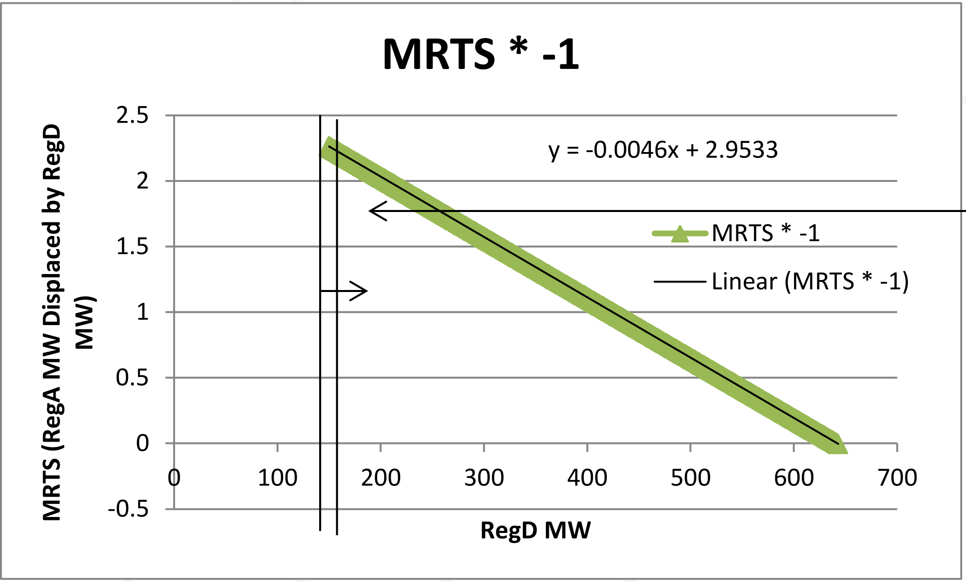
PJM based combinations: MRTS

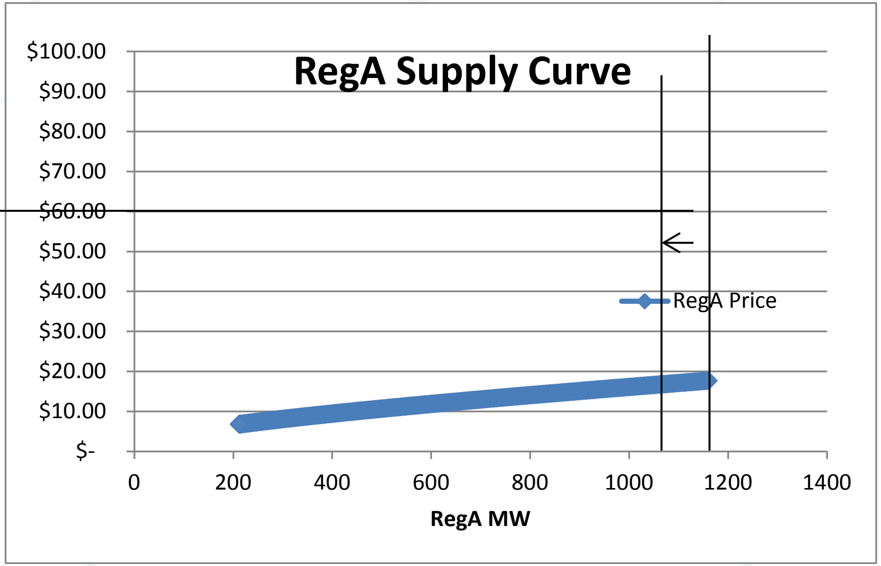


Demand curve for RegD in optimization is determined by calculating the marginal displaced cost of RegA for each MW of D

Monitoring Analytics

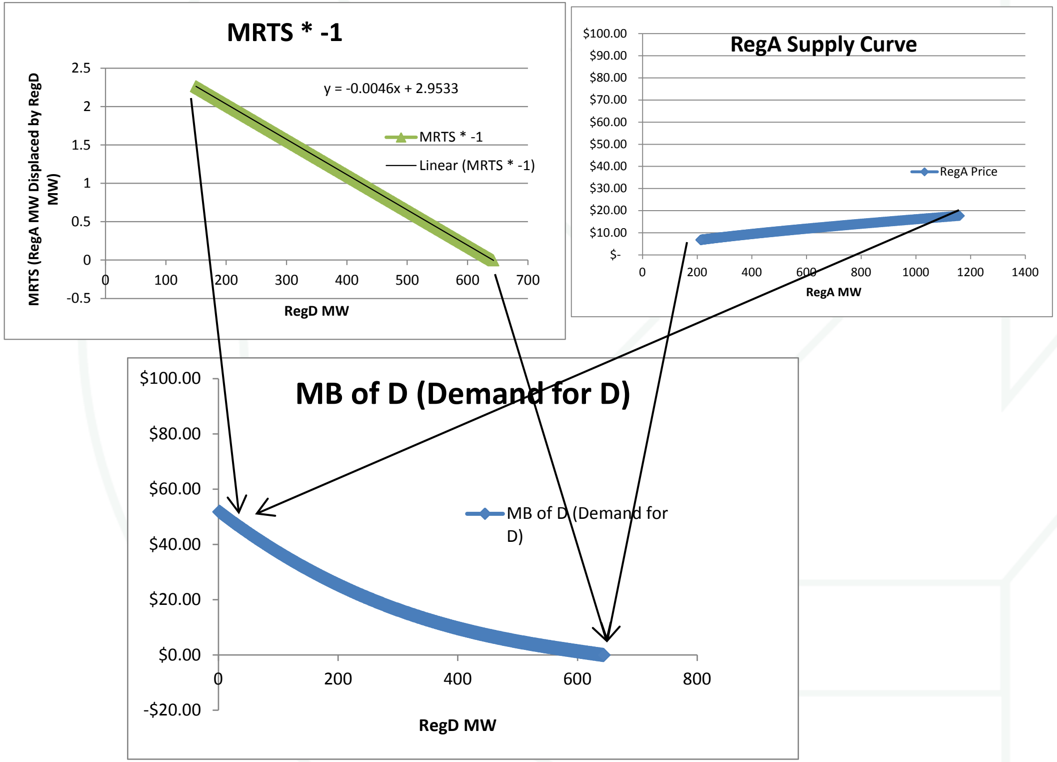
PJM based combinations: MRTS





If MRTS = 2, then 1 MW of RegD shifts demand of RegA to the left by 2 MW, moving along isoquant. Cheapest D replaces most expensive A (moving from 100% A to less than 100% A).

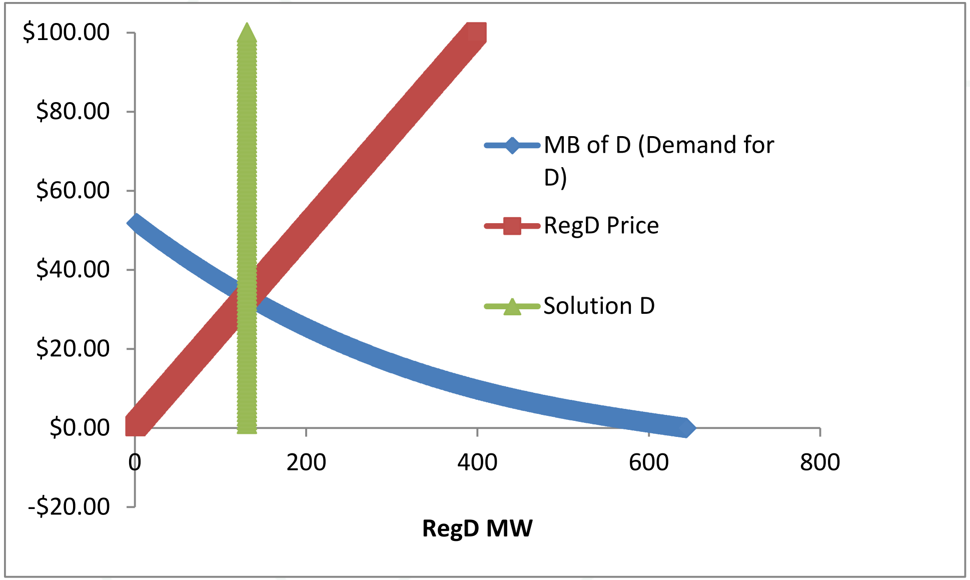
PJM based combinations: MRTS



RegD demand curve (MB of D MW) is calculated in terms of the $/MRTS of using D MW to displace A MW.

Determined by calculating the marginal displaced cost of RegA for each MW of D

PJM based combinations: MRTS

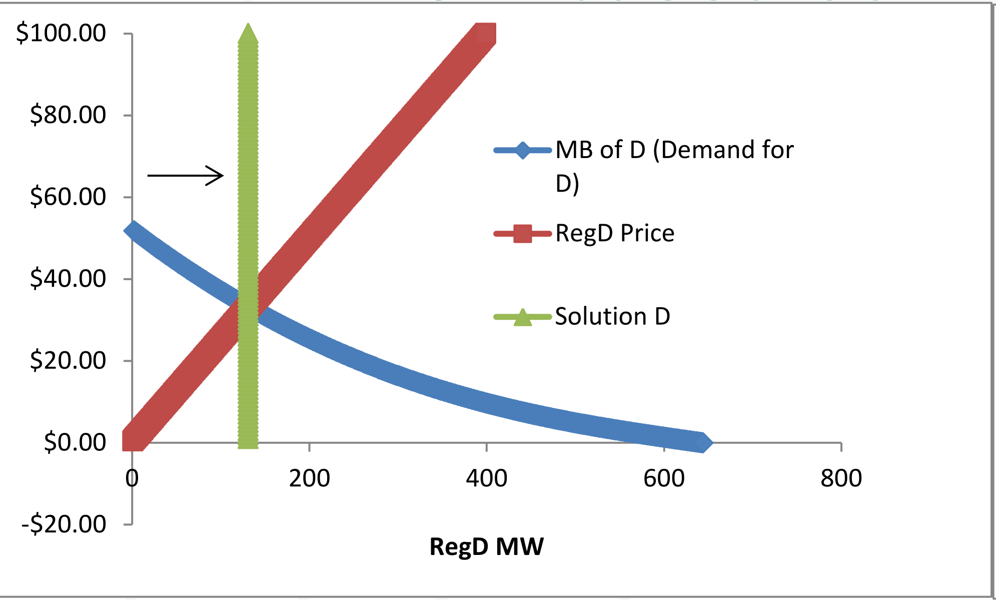


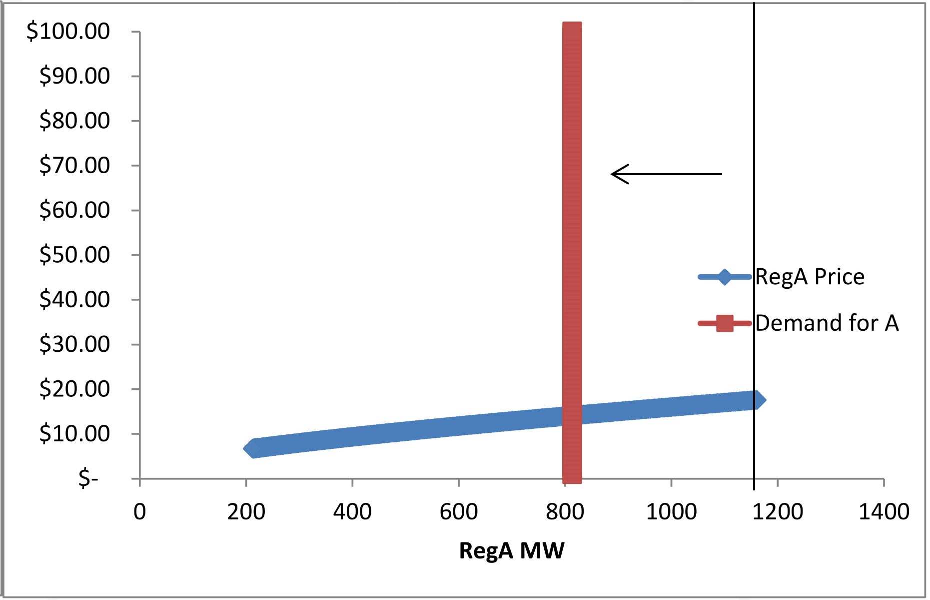
Optimal level of D MW:

Where Demand for D (MB of displaced A holding control constant) = MC of D.

Point of intersection between the demand for D MW (MB of D) and the MC of D MW (supply).

PJM based combinations: MRTS





Each MW of D reduces the demand for Reg A MW based on MRTS (and vice versa). Resulting combination is on the PJM isoquant.

Consistent Application of MRTS

- Single clearing price (input) model.
- Resources evaluated and paid on marginal effective MW basis.
- MRTS converts offers into equivalent units
  - MRTS of A = 1, MRTS of D = MRTS (MW D)
- P = marginal price of Effective MW, highest cost cleared resource (A or D), in terms of $/RegA equivalent.
  - P = Max(MAX(PD (MW D) /MRTS), MAX(PA(MW A))
- Payment is per marginal RegA equivalent MW.
  - Payment = P x MRTS x MW

Monitoring Analytics, LLC
2621 Van Buren Avenue
Suite 160
Eagleville, PA
19403
610) 271-8050
MA@monitoringanalytics.com
www.MonitoringAnalytics.com