

Regulation Market Optimization

RMISTF

November 16, 2016

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Monitoring Analytics

Basics of approach: Isoquant

- **Isoquant:**
- **Set of points that defines combinations of inputs that provide a fixed output. Shows that the output is a defined function of the two different inputs.**
- **Regulation Isoquant:**
- **Set of combinations of RegD MW and RegA MW that provide an expected level of ACE control.**

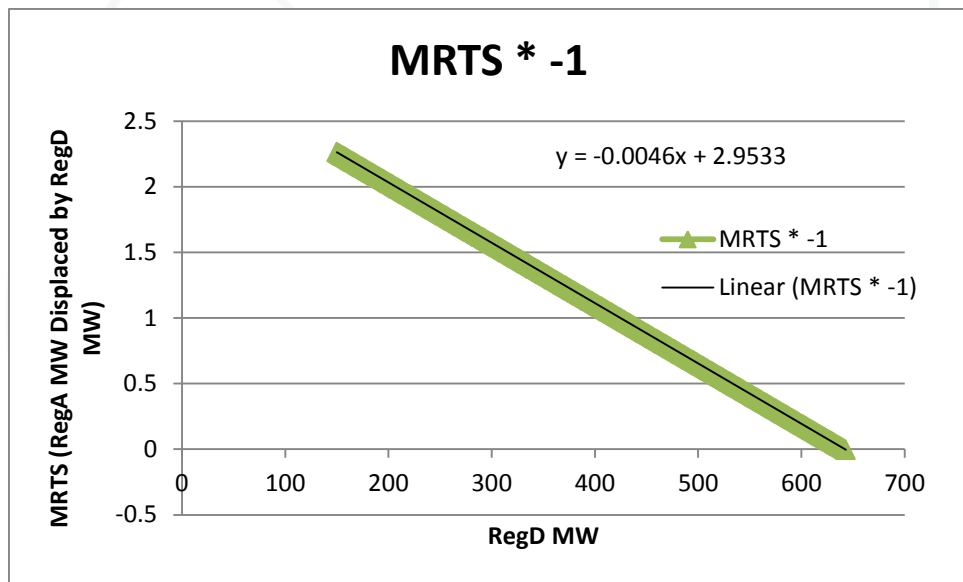
PJM Modeled Control Scores for various new signal based RegA/RegD combinations

Average of Con RegA RegD	0	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1000
0									260.4676	253.52	248.3677	244.6573	242.2628	240.7543	239.9615	239.7899	240.2729	241.2953	242.7395	244.6651	246.8473
50								238.2629	229.0765	222.6455	218.013	214.7082	212.4433	211.1378	210.4281	210.2994	210.8478	211.7306	212.9072	214.3292	
100							228.2232	215.4021	205.4934	198.2	192.9836	188.9761	185.8499	184.129	183.0455	182.5161	182.5368	182.6374	182.8033		
150					228.0722	209.5654	195.4245	184.7992	176.5644	170.5145	165.8044	161.941	159.2971	157.4591	156.1529	155.3979	155.1371				
200				233.9134	212.0164	193.1807	178.1102	166.6785	157.9115	151.3098	145.6437	141.1636	138.1651	135.5467	133.5958	132.2064					
250			245.6584	219.5599	197.3763	178.6492	163.3712	151.3158	142.426	135.1577	129.2936	124.6338	121.2239	118.0783	115.8994						
300		263.4692	234.0175	207.258	184.5939	165.7654	150.6714	138.4881	129.8193	122.4056	116.7235	112.1725	108.7225	105.608							
350		286.7793	253.7045	223.8123	196.4878	173.5508	154.8205	140.0527	128.2034	119.6716	112.7312	107.2895	103.2071	99.75885							
400	314.2612	278.4291	245.054	214.5861	186.9844	164.0033	145.4262	131.4191	120.2336	112.18	105.4005	100.4343	96.46236								
450	306.7571	271.0234	236.9429	206.1838	178.5909	155.6877	137.8448	124.4198	114.2869	106.3168	100.141	95.40625									
500	300.1569	264.2888	229.7802	198.5891	171.247	148.3857	131.3197	118.7627	109.2292	101.869	96.14533										
550	294.1045	258.0281	223.5786	191.8847	164.5043	142.1707	126.0324	113.9991	105.1491	98.19274											
600	288.4192	252.372	217.5941	186.0489	158.6807	137.0086	121.5723	110.2575	101.8961												
650	282.9796	247.1173	212.1962	180.5825	153.5373	132.6016	117.9852	107.0773													
700	277.8865	242.4719	207.3627	175.6695	148.7507	128.9552	114.7422														
750	273.279	237.9201	202.9188	171.631	144.5769	125.6129															
800	268.7797	233.5976	198.6674	167.6297	141.1609																
850	264.3141	229.5828	194.8414	163.839																	
900	260.223	225.735	191.2823																		
950	256.4235	222.0159																			
1000	252.7491																				

Basics of approach: MRTS

- **MRTS = Marginal Rate of Technical Substitution.**
- **The slope of the isoquant at any point (where a point is a combination of inputs) for a specific level of fixed output. Defines the marginal rate of substitution between inputs at each point.**
- **The rate of substitution between inputs holding output constant.**
- **An exchange rate that converts substitutable inputs into common units so that they can be compared directly in optimization and in the market.**

PJM based combinations: MRTS



MRTS = Point specific slopes of the isoquant defining the rate of substitution.

Derivative of curve defining combinations of RegA/RegD

Basics of approach: MRTS

- **MRTS: The marginal rate of substitution between RegD and RegA**

- **Example:**

$$\text{MRTS} = (\text{MRTS of D MW for A MW}) = 2.$$

- **Indicates that at this point on the isoquant:**
 - **1 D MW can be substituted (1 MW D x MRTS = 2) for 2 MW of A at that point on the isoquant.**

OR

- **2 MW of A can be substituted (2 MW D/MRTS = 1) for 1 MW of D at that point on the isoquant.**

Basics of approach: MRTS as exchange rate

- Using MRTS a RegD offer can be compared directly to a RegA offer.
 - If $\text{MRTS} = (\text{MRTS of D MW for A MW}) = 2$.
 - $(\$20/\text{MW D}) / \text{MRTS} = \text{offer in terms of } \$/\text{MW A}$
 - $(\$20 \text{ per RegD MW}) / 2 = \$10/\text{MW}$ in terms of equivalent A MW.
- Defines whether it is economic to exchange 1 MW of D for $\text{MRTS} * \text{MW of A}$ or $(\text{A MW})/\text{MRTS}$ for 1 MW of D .
- Basis of the decision at any point is based on the marginal relative values in terms of output and price at that point.

Consistent Application of MRTS

- Single clearing price (input) model.
- Resources evaluated and paid on per marginal effective MW basis.
- MRTS converts offers into equivalent units
 - MRTS of A = 1, MRTS of D = MRTS (MW D)
- P = marginal price of Effective MW, highest cost cleared resource (A or D), in terms of \$/RegA equivalent.
 - $P = \text{Max}(\text{MAX}(\text{PD (MW D) / MRTS}), \text{MAX}(\text{PA(MW A)})$
- Payment is per marginal RegA equivalent MW.
 - $\text{Payment} = P \times \text{MRTS} \times \text{MW}$

Example of Market Optimization

November 9, 2016

Howard Haas

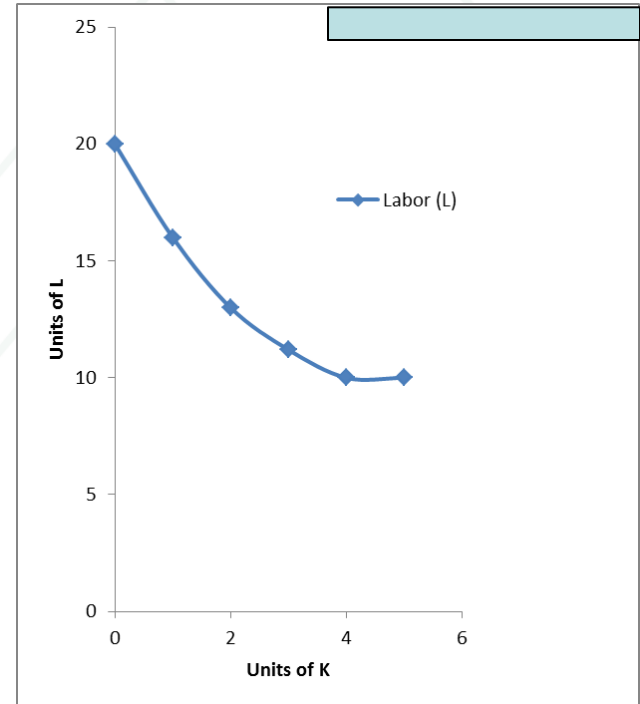


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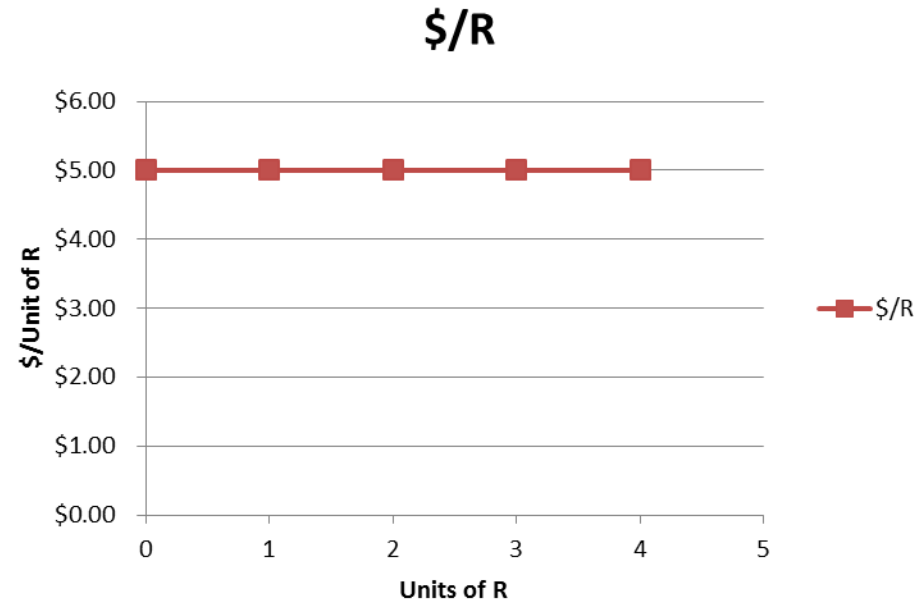
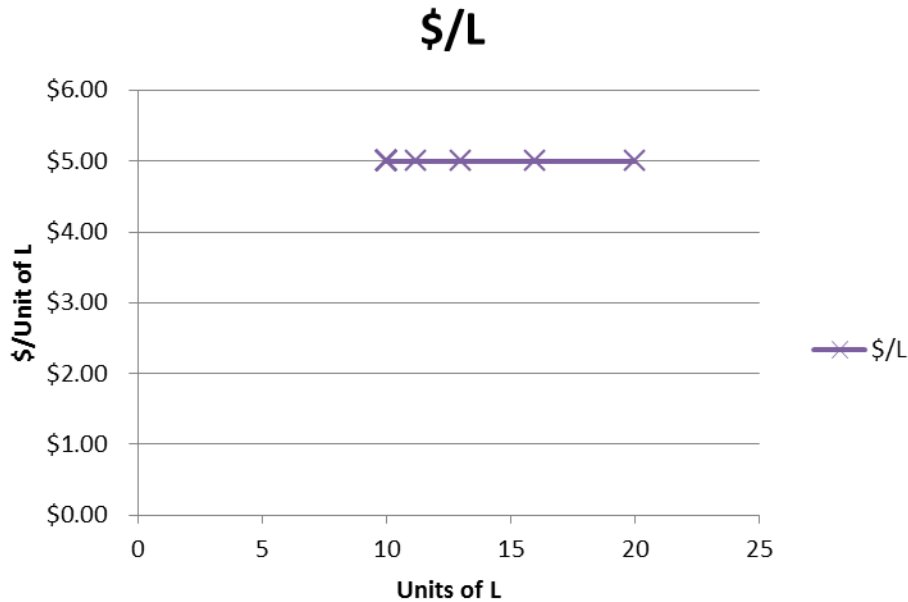
Microeconomics: Isoquant

Isoquant			
Cars/H	L/HR	R/HR	MRTS (Change in L/Change in R)
1.00	20	0	NA
1.00	16	1	4
1.00	13	2	3
1.00	11.2	3	1.8
1.00	10	4	1
1.00	10	5	0

What is optimal?



Need Supply Curves to Determine Optimal



Basics of approach: Two input production model

Least cost combination

Isoquant				MRTS (Change in L/Change in R)	\$/L	\$/R	Total Cost
Cars/H	L/HR	R/HR					
1.00	20	0	NA	\$5.00	\$5.00	\$100	
1.00	16	1	4	\$5.00	\$5.00	\$85	
1.00	13	2	3	\$5.00	\$5.00	\$75	
1.00	11.2	3	1.8	\$5.00	\$5.00	\$71	
1.00	10	4	1	\$5.00	\$5.00	\$70	
1.00	10	5	0	\$5.00	\$5.00	\$75	



Price information needed

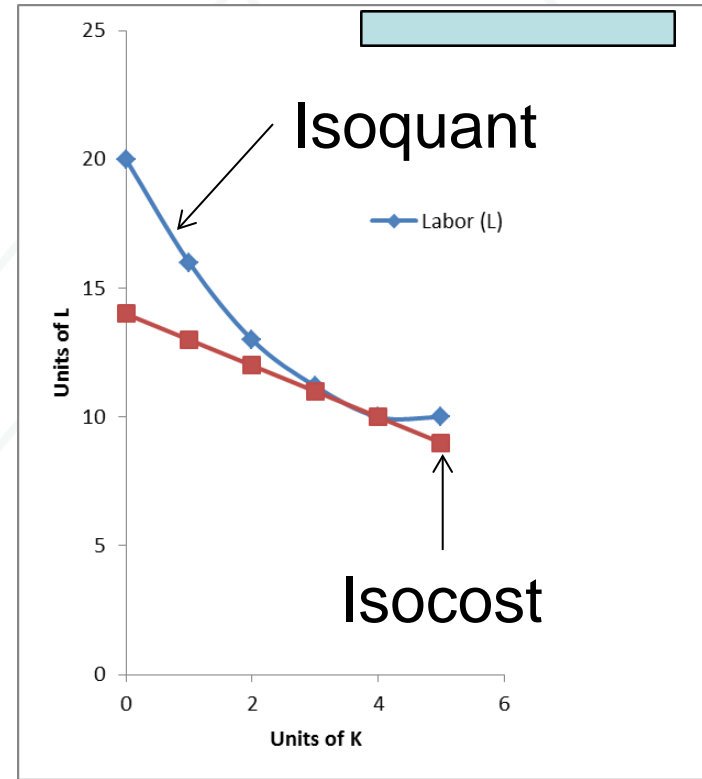
Basics of approach: Two input production model

Isoquant				MRTS (Change in L/Change in R)	\$/L	\$/R
Cars/H	L/HR	R/HR				
1.00	20	0	NA	\$5.00	\$5.00	
1.00	16	1	4	\$5.00	\$5.00	
1.00	13	2	3	\$5.00	\$5.00	
1.00	11.2	3	1.8	\$5.00	\$5.00	
1.00	10	4	1	\$5.00	\$5.00	
1.00	10	5	0	\$5.00	\$5.00	

Where slope of prices = MRTS,
Least cost combination

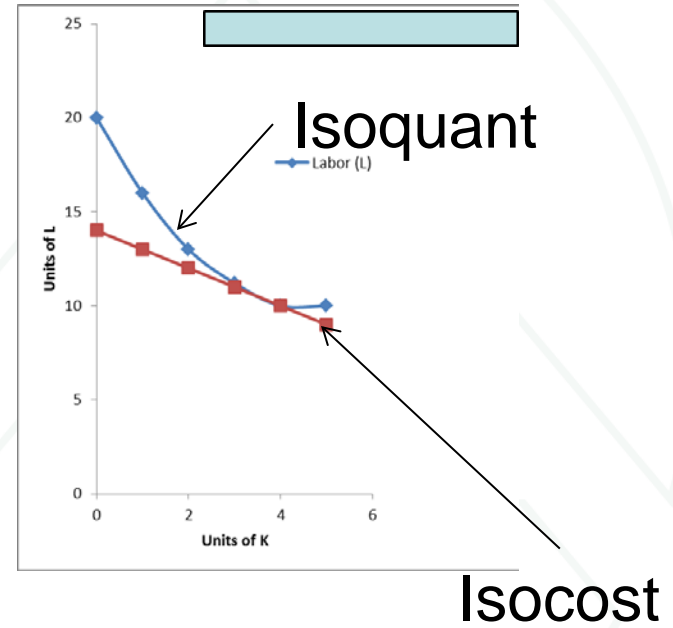
Basics of approach: Two input production model

Isoquant				MRTS (Change in L/Change in R)	\$/L	\$/R	Slope of Isocost (Ratio of Prices)
Cars/H	L/HR	R/HR					
1.00	20	0	NA	\$5.00	\$5.00	1.00	
1.00	16	1	4	\$5.00	\$5.00	1.00	
1.00	13	2	3	\$5.00	\$5.00	1.00	
1.00	11.2	3	1.8	\$5.00	\$5.00	1.00	
1.00	10	4	1	\$5.00	\$5.00	1.00	
1.00	10	5	0	\$5.00	\$5.00	1.00	



Basics of approach: Two input production model

Isoquant				MRTS (Change in L/Change in R)	\$/L	\$/R	Slope of Isocost (Ratio of Prices)
Cars/H	L/HR	R/HR					
1.00	20	0	NA	\$5.00	\$5.00	1.00	
1.00	16	1	4	\$5.00	\$5.00	1.00	
1.00	13	2	3	\$5.00	\$5.00	1.00	
1.00	11.2	3	1.8	\$5.00	\$5.00	1.00	
1.00	10	4	1	\$5.00	\$5.00	1.00	
1.00	10	5	0	\$5.00	\$5.00	1.00	



- Where ratio of prices = MRTS, all resources paid the same per effective output contribution at market solution

Basics of approach: Two input production model

Isoquant							
Cars/H	L/HR	R/HR	MRTS (Change in L/Change in R)	\$/L	\$/R	Slope of Isocost (Ratio of Prices)	(\$/R)/MRTS
1.00	20	0	NA	\$5.00	\$5.00	1.00	
1.00	16	1	4	\$5.00	\$5.00	1.00	\$ 1.25
1.00	13	2	3	\$5.00	\$5.00	1.00	\$ 1.67
1.00	11.2	3	1.8	\$5.00	\$5.00	1.00	\$ 2.78
1.00	10	4	1	\$5.00	\$5.00	1.00	\$ 5.00
1.00	10	5	0	\$5.00	\$5.00	1.00	NA

$\$L > \$R/MRTS$

$\$L = \$R/MRTS$

$\$L < \$R/MRTS$

- Where ratio of prices = MRTS, all resources paid the same per effective contribution at market solution
- \$5 per L = \$5 per R/MRTS

Basics of approach: Two input production model

Isoquant							
Cars/H	L/HR	R/HR	MRTS (Change in L/Change in R)	\$/L	\$/R	Slope of Isocost (Ratio of Prices)	(\$/R)/MRTS
1.00	20	0	NA	\$5.00	\$5.00	1.00	
1.00	16	1	4	\$5.00	\$5.00	1.00	\$ 1.25
1.00	13	2	3	\$5.00	\$5.00	1.00	\$ 1.67
1.00	11.2	3	1.8	\$5.00	\$5.00	1.00	\$ 2.78
1.00	10	4	1	\$5.00	\$5.00	1.00	\$ 5.00
1.00	10	5	0	\$5.00	\$5.00	1.00	NA

$\$L = \R / MRTS

- **\$5 per L = \$5 per R/MRTS**
- **All resources paid the same price in equilibrium, the same price per common unit at the margin (market solution).**

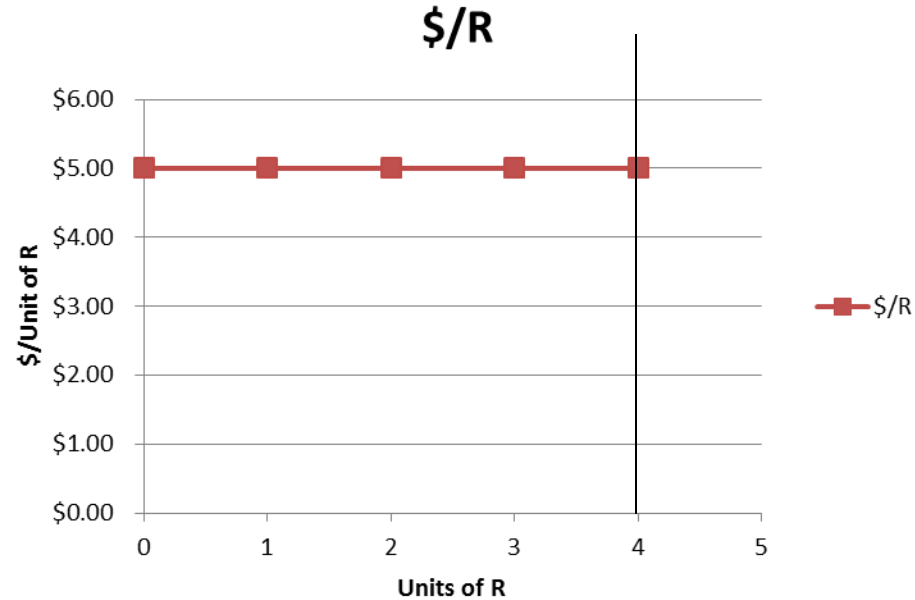
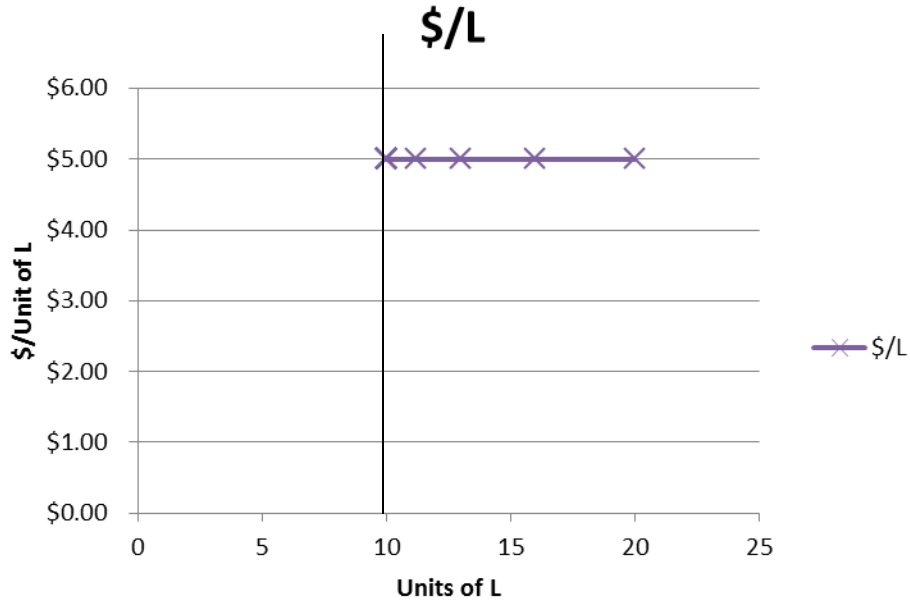
Basics of approach: Two input production model

Isoquant			MRTS (Change in L/Change in R)			Slope of Isocost (Ratio of Prices)	(\$/R)/MRTS
Cars/H	L/HR	R/HR		\$/L	\$/R		
1.00	20	0	NA	\$5.00	\$5.00	1.00	
1.00	16	1	4	\$5.00	\$5.00	1.00	\$ 1.25
1.00	13	2	3	\$5.00	\$5.00	1.00	\$ 1.67
1.00	11.2	3	1.8	\$5.00	\$5.00	1.00	\$ 2.78
1.00	10	4	1	\$5.00	\$5.00	1.00	\$ 5.00
1.00	10	5	0	\$5.00	\$5.00	1.00	NA

$\$L = \R / MRTS

- **\$5 per L = \$5 per R/MRTS = \$5 per unit L**
- **Each unit of L gets \$5.**
- **Each unit of R gets \$5 per unit of L equivalent = \$5 x MRTS x R**

Supply and Demand



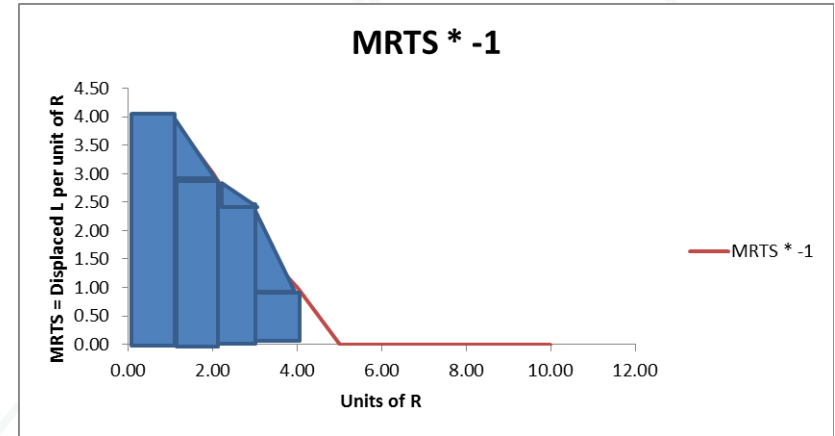
Basics of approach: MRTS as exchange rate

Effective	L	L	R	MRTS
20	20.00	0.00	0.00	
20	16.00	1.00	4.00	
20	13.00	2.00	3.00	
20	11.20	3.00	1.80	
20	10.00	4.00	1.00	
20	10.00	5.00	0.00	
20	10.00	6.00	0.00	
20	10.00	7.00	0.00	
20	10.00	8.00	0.00	
20	10.00	9.00	0.00	
20	10.00	10.00	0.00	

- **MRTS = Change in L/Change in R, holding output constant.**
- **MRTS translates units of R into effective units of L on the margin.**
 - **(1 unit of R * MRTS) = marginal substitution for L**
 - **Total displacement of L by R at any point can be calculated as area under the MRTS curve defined in change in L for change in R.**

Basics of approach: Staying on the curve

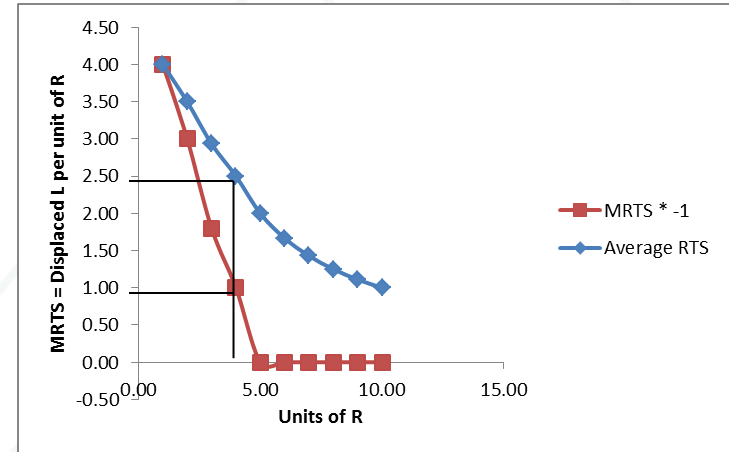
Effective L	L	R	MRTS	Area under Curve Calculation of effective L from R (Displaced L from R)	Residual L	Total Effective L
20	20.00	0.00	0.00		20.00	20.000
20	16.00	1.00	4.00	4.00	16.00	20.000
20	13.00	2.00	3.00	7.00	13.00	20.000
20	11.20	3.00	1.80	8.80	11.20	20.000
20	10.00	4.00	1.00	10.00	10.00	20.000
20	10.00	5.00	0.00	10.00	10.00	20.000
20	10.00	6.00	0.00	10.00	10.00	20.000
20	10.00	7.00	0.00	10.00	10.00	20.000
20	10.00	8.00	0.00	10.00	10.00	20.000
20	10.00	9.00	0.00	10.00	10.00	20.000
20	10.00	10.00	0.00	10.00	10.00	20.000



- AT 4 R, Displacing Area in terms of L, holding output constant.
- Producing the equivalent of 20 units of L using 4 R and 10 L
- Just confirms that the combination of 10 L and 4 R = output of 20 L = 1 Car

MRTS not the ARTS explains the contribution at solution point

Effective L	L	R	MRTS	Area under Curve Calculation of effective L from R (Displaced L from R)	Residual L	Total Effective L	Average RTS
20	20.00	0.00	0.00		20.00	20.000	
20	16.00	1.00	4.00	4.00	16.00	20.000	4
20	13.00	2.00	3.00	7.00	13.00	20.000	3.5
20	11.20	3.00	1.80	8.80	11.20	20.000	2.933333
20	10.00	4.00	1.00	10.00	10.00	20.000	2.5
20	10.00	5.00	0.00	10.00	10.00	20.000	2
20	10.00	6.00	0.00	10.00	10.00	20.000	1.666667
20	10.00	7.00	0.00	10.00	10.00	20.000	1.428571
20	10.00	8.00	0.00	10.00	10.00	20.000	1.25
20	10.00	9.00	0.00	10.00	10.00	20.000	1.111111
20	10.00	10.00	0.00	10.00	10.00	20.000	1



MRTS not the ARTS explains the contribution

Effective	L	R	MRTS	Average	RTS	(\$/R)/MR	TS
L	L	R	MRTS	Average	RTS	\$/L	\$/R
20	20.00	0.00	0.00			\$5.00	\$5.00
20	16.00	1.00	4.00	4		\$5.00	\$5.00 \$ 1.25
20	13.00	2.00	3.00	3.5		\$5.00	\$5.00 \$ 1.67
20	11.20	3.00	1.80	2.933333		\$5.00	\$5.00 \$ 2.78
20	10.00	4.00	1.00	2.5		\$5.00	\$5.00 \$ 5.00
20	10.00	5.00	0.00	2		\$5.00	\$5.00 NA

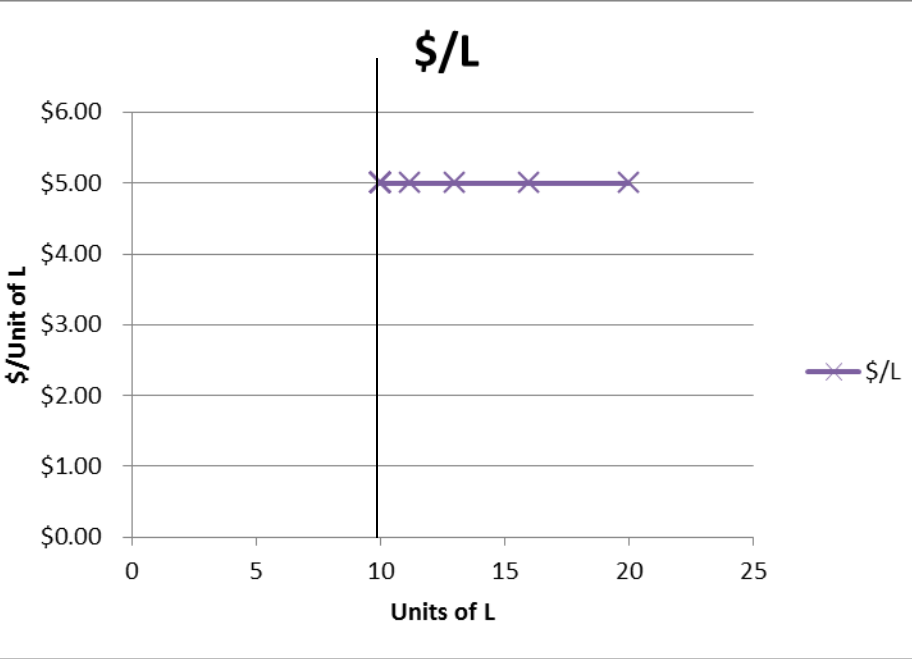
- \$5 per average effective L?
- Clearing price per unit of L is \$5.
- Each unit of R would get more per effective L than L

- **\$5 x MRTS x R = \$5/effective unit of L ← correct**
- **\$5 x ARTS X R = \$12.5/effective unit of L ← wrong**

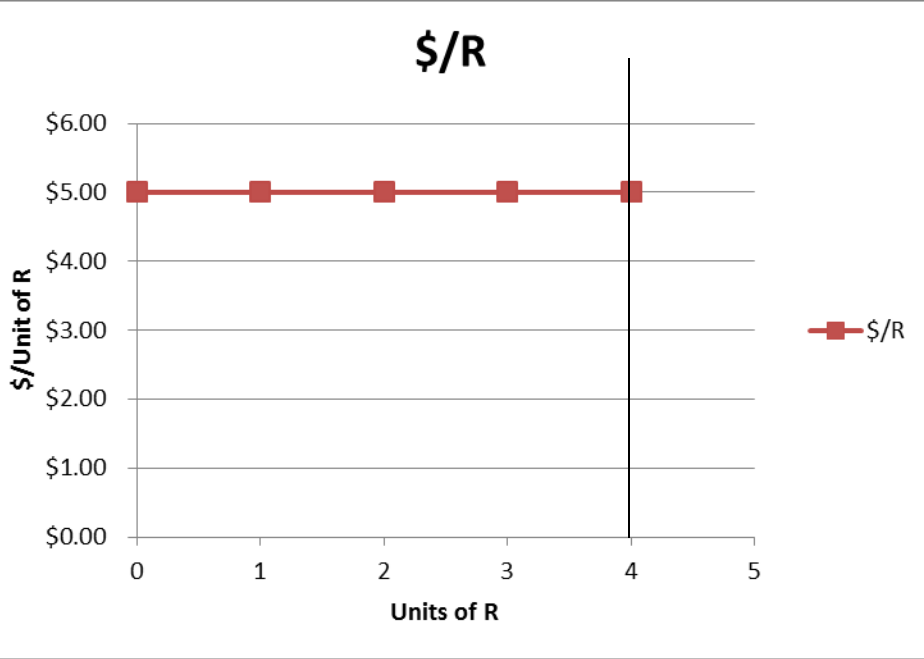
Supply and Demand

\$12.5?

\$/L



\$/R



Consistent Application of MRTS

- Single clearing price (input) model.
- Resources evaluated and paid on marginal effective MW basis.
- MRTS converts offers into equivalent units
 - MRTS of A = 1, MRTS of D = MRTS (MW D)
- P = marginal price of Effective MW, highest cost cleared resource (A or D), in terms of \$/RegA equivalent.
 - $P = \text{Max}(\text{MAX}(\text{PD (MW D) / MRTS}), \text{MAX}(\text{PA(MW A)})$
- Payment is per marginal RegA equivalent MW.
 - $\text{Payment} = P \times \text{MRTS} \times \text{MW}$

Quiz

What is the optimal mix?

How much does a unit of L get paid at the optimal mix?

How much does a unit of R get paid at the optimal mix?

How much does a unit of R get paid in terms of effective unit of L at the optimal mix?

Isoquant					
Cars/H	L/HR	R/HR	MRTS (Change in L/Change in R)	\$/L	\$/R
1.00	20	0	NA	\$1.25	\$5.00
1.00	16	1	4	\$1.25	\$5.00
1.00	13	2	3	\$1.25	\$5.00
1.00	11.2	3	1.8	\$1.25	\$5.00
1.00	10	4	1	\$1.25	\$5.00

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