Electrical Theory

Mathematics Review

**PJM State & Member Training Dept.**

Objectives

By the end of this presentation the Learner should be able to:

- Use the basics of trigonometry to calculate the different components of a right triangle

- Compute Per-Unit Quantities

- Identify the two components of Vectors

Right Triangles

Mathematics Review

- To be able to understand basic AC power concepts, a familiarization with the relationships between the angles and sides of a right triangle is essential

- A right triangle is defined as a triangle in which one of the three angles is equal to 90°

Mathematics Review
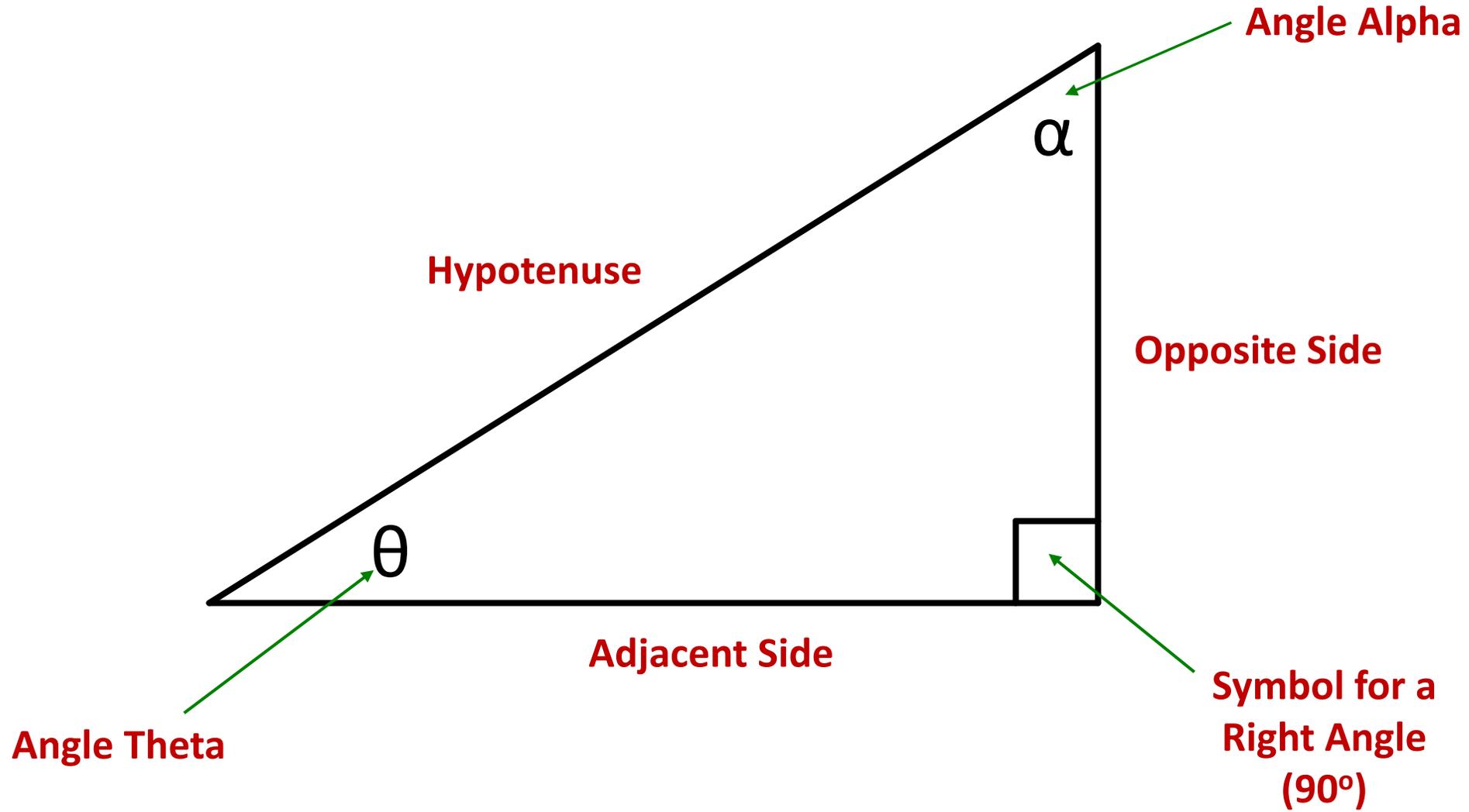
Mathematics Review

- Given the lengths of two sides of a right triangle, the third side can be determined using the Pythagorean Theorem

$$\text{Hypotenuse}^2 = \text{Opposite}^2 + \text{Adjacent}^2$$

Example:

A rope stretches from the top of one pole 50 feet high to the top of another pole 20 feet high, standing 16 feet away.

How long is the rope?

30 ft.

???

50 ft.

20 ft.

16 ft.
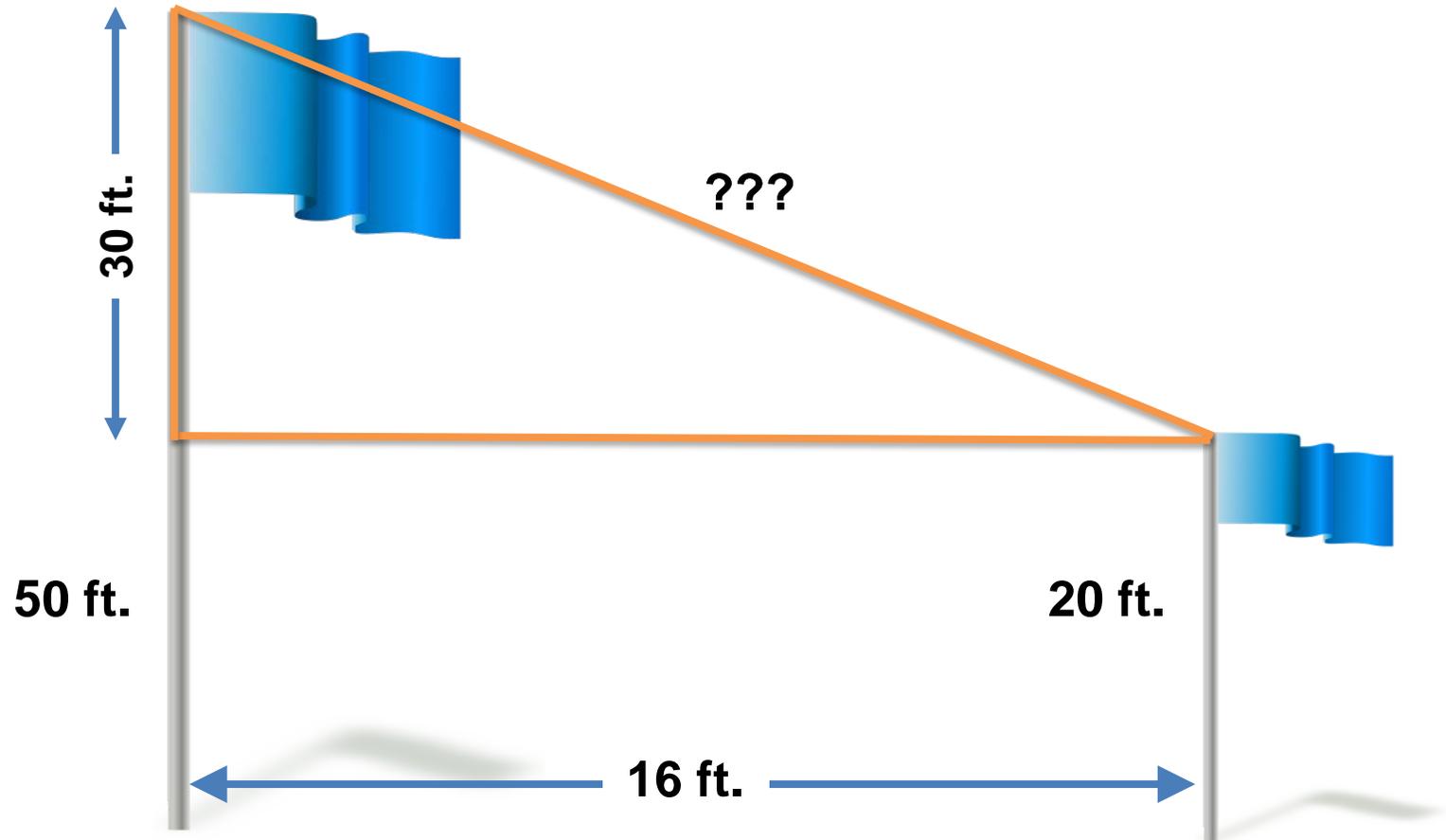
Example:
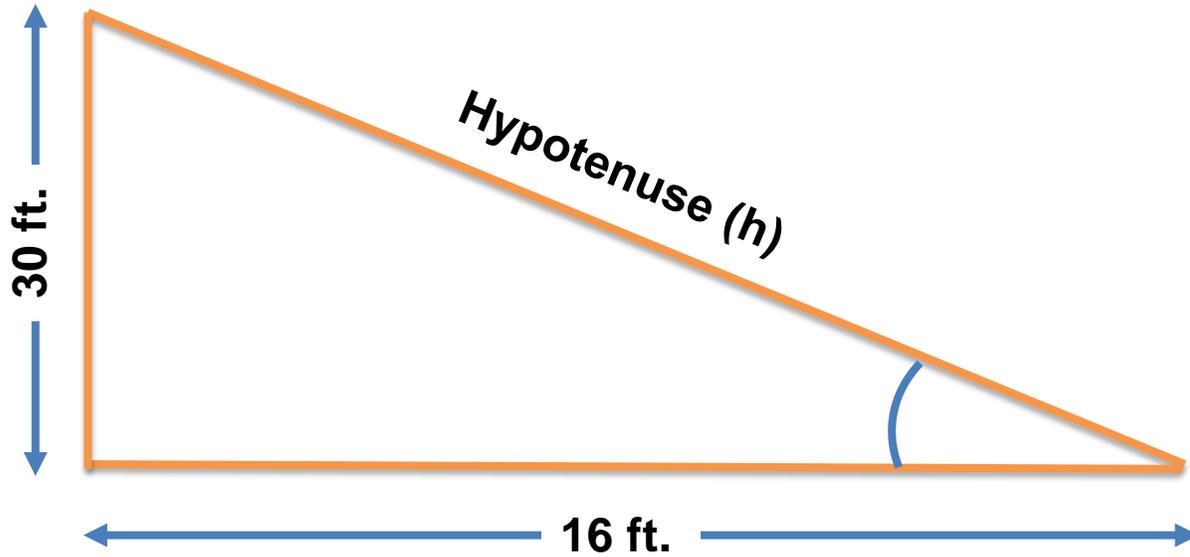
$$h^2 = a^2 + o^2$$
$$h^2 = 16^2 + 30^2$$
$$h^2 = 256 + 900$$
$$h^2 = 1156$$
$$h = \sqrt{1156}$$
$$h = 34$$

Mathematics Review

- Once the sides are known, the next step in solving the right triangle is to determine the two unknown angles of the right triangle

- All of the angles of any triangle always add up to 180°

- In solving a right triangle, the remaining two unknown angles must add up to 90°

- Basic trigonometric functions are needed to solve for the values of the unknown angles

Trigonometry

Mathematics Review

- The sine function is a periodic function in that it continually repeats itself
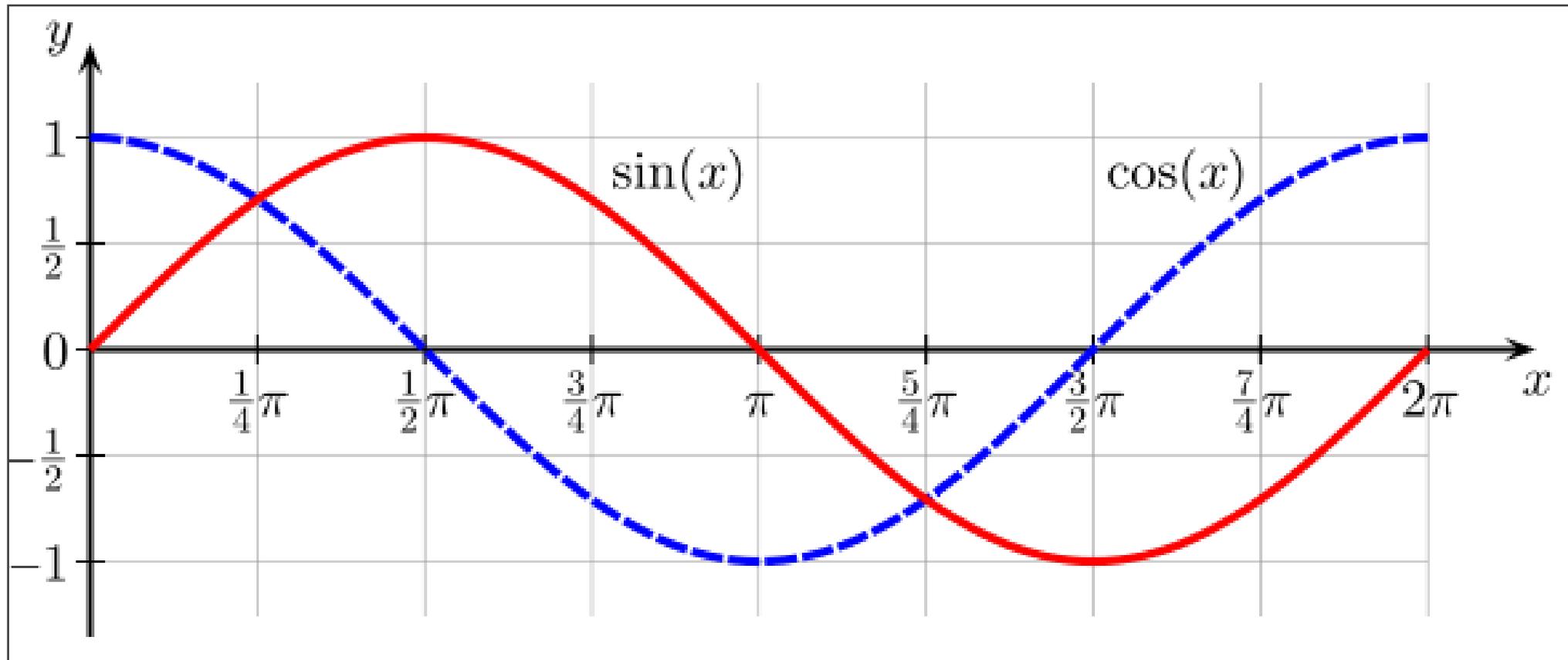
Mathematics Review

- In order to solve right triangles, it is necessary to know the value of the sine function between 0° and 90°

- Sine of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the hypotenuse

SIN θ = Opposite Side / Hypotenuse
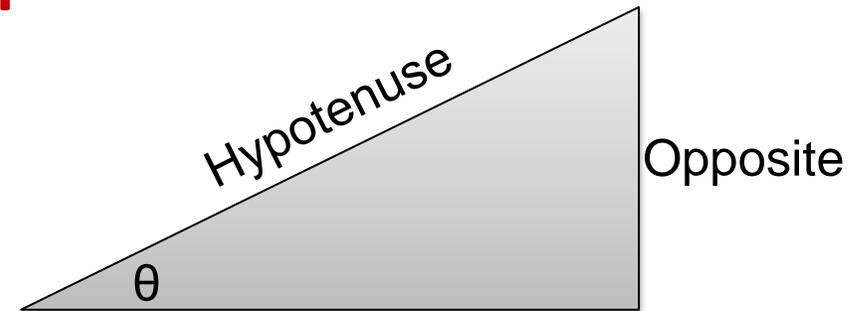
Mathematics Review

- Cosine function is a periodic function that is identical to the sine function except that it leads the sine function by 90°
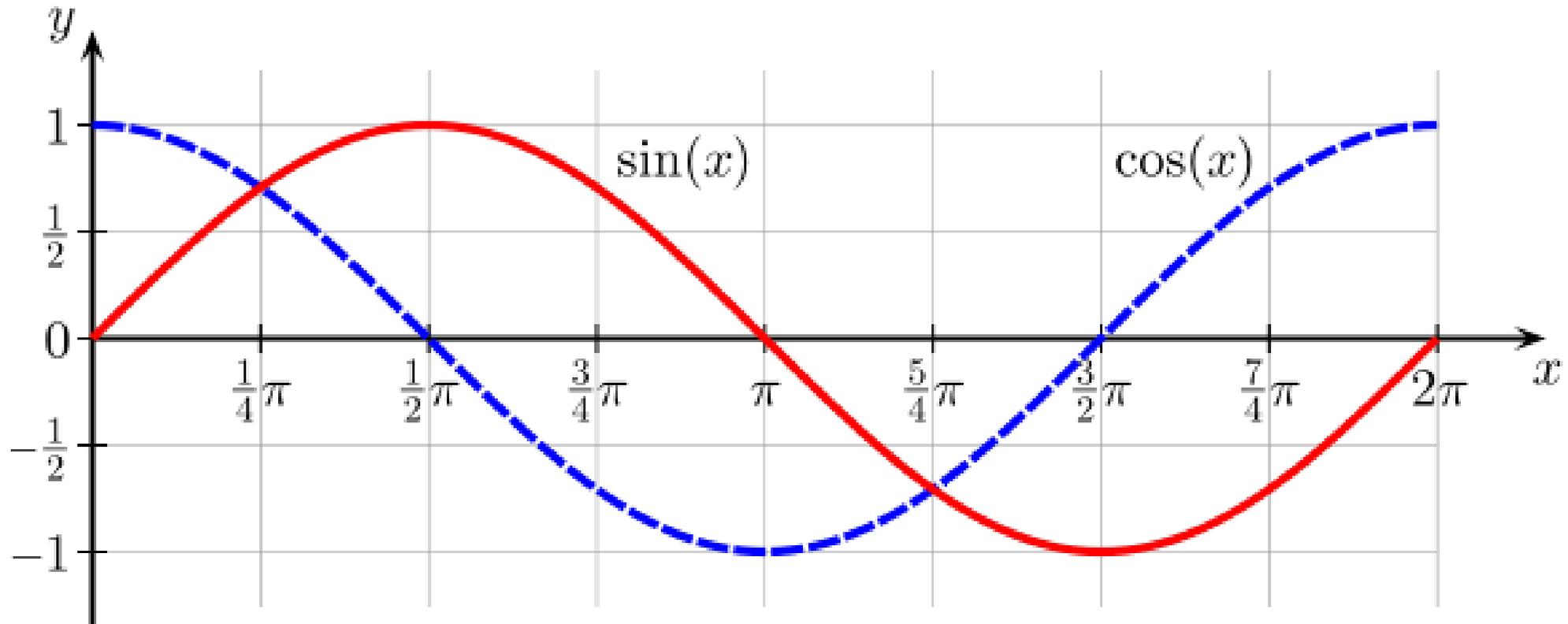
Mathematics Review

- As an example, the cosine function at 0° is 1 whereas the sine function does not reach the value of 1 until 90°

- Cosine function of either of the unknown angles of a right triangle is the ratio of the length of the adjacent side to the length of the hypotenuse

COS θ = Adjacent Side / Hypotenuse
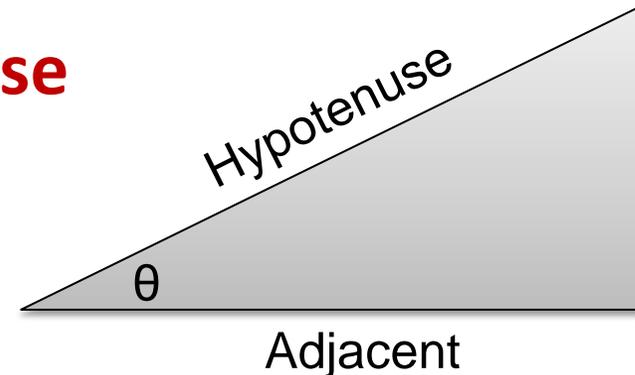
Mathematics Review

- The tangent function of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the adjacent side

TAN θ = Opposite Side / Adjacent Side
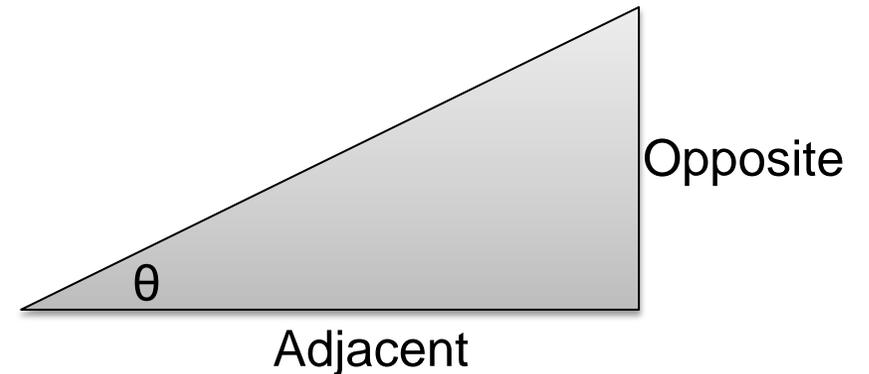
Mathematics Review

Example:

- Given: Side H = 5, Side A = 4

- Find: Side O, Angle θ and Angle α
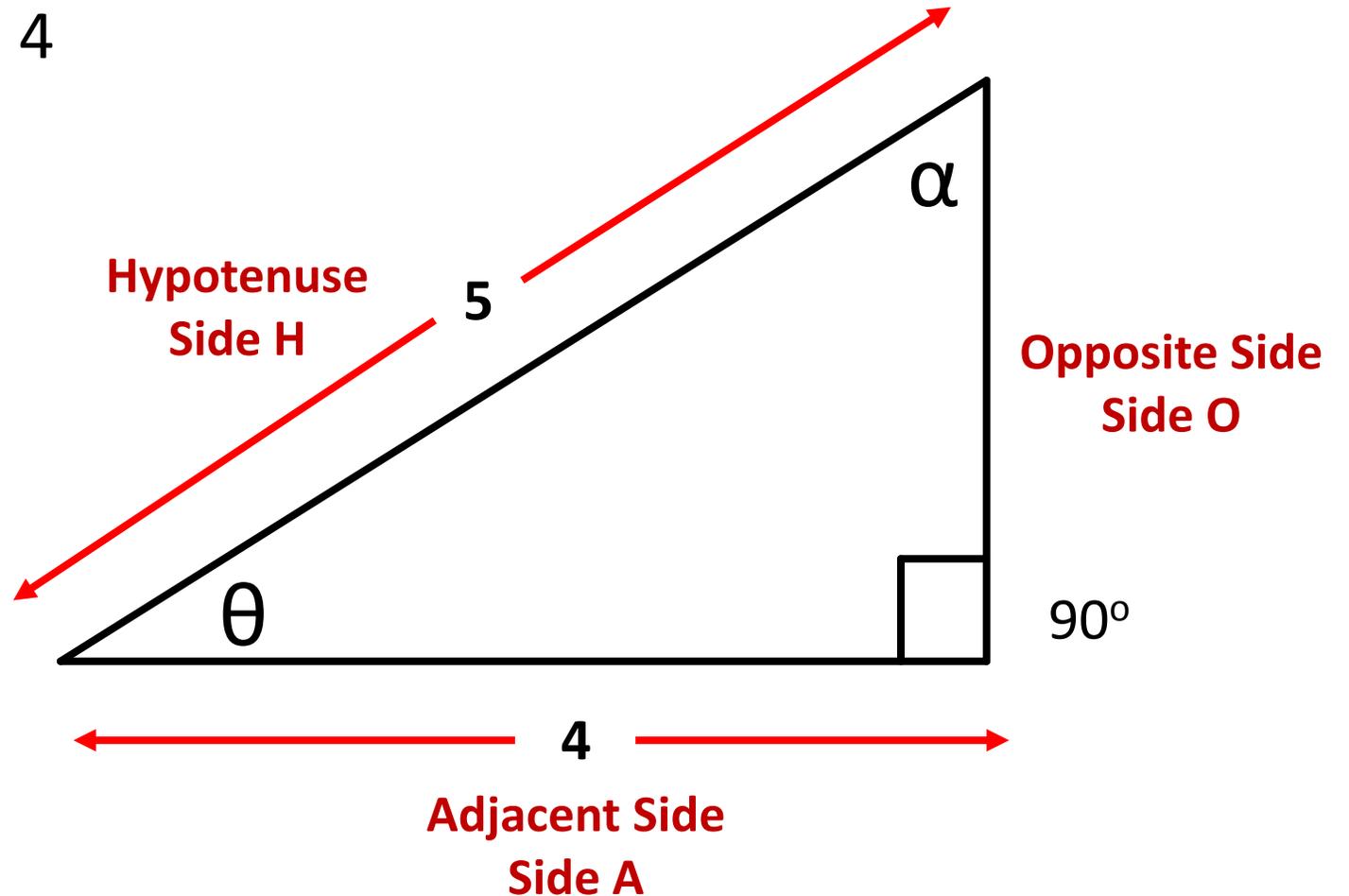
Mathematics Review

- Find Side O:
$$H^2 = A^2 + O^2$$
$$25 = 16 + O^2$$
$$25 - 16 = O^2$$
$$\sqrt{25 - 16} = O$$
$$O = \sqrt{9} = 3$$

- Find θ:
$$\cos \theta = \frac{A}{H} = \frac{4}{5} = .8$$
$$\cos^{-1}(.8) = 36.87°$$

- Find α:
$$180° - 90° - 36.87° = 53.13°$$

$$\alpha = 53.13°$$

α

H = 5

O = 3

θ = 36.87°

90°

A = 4
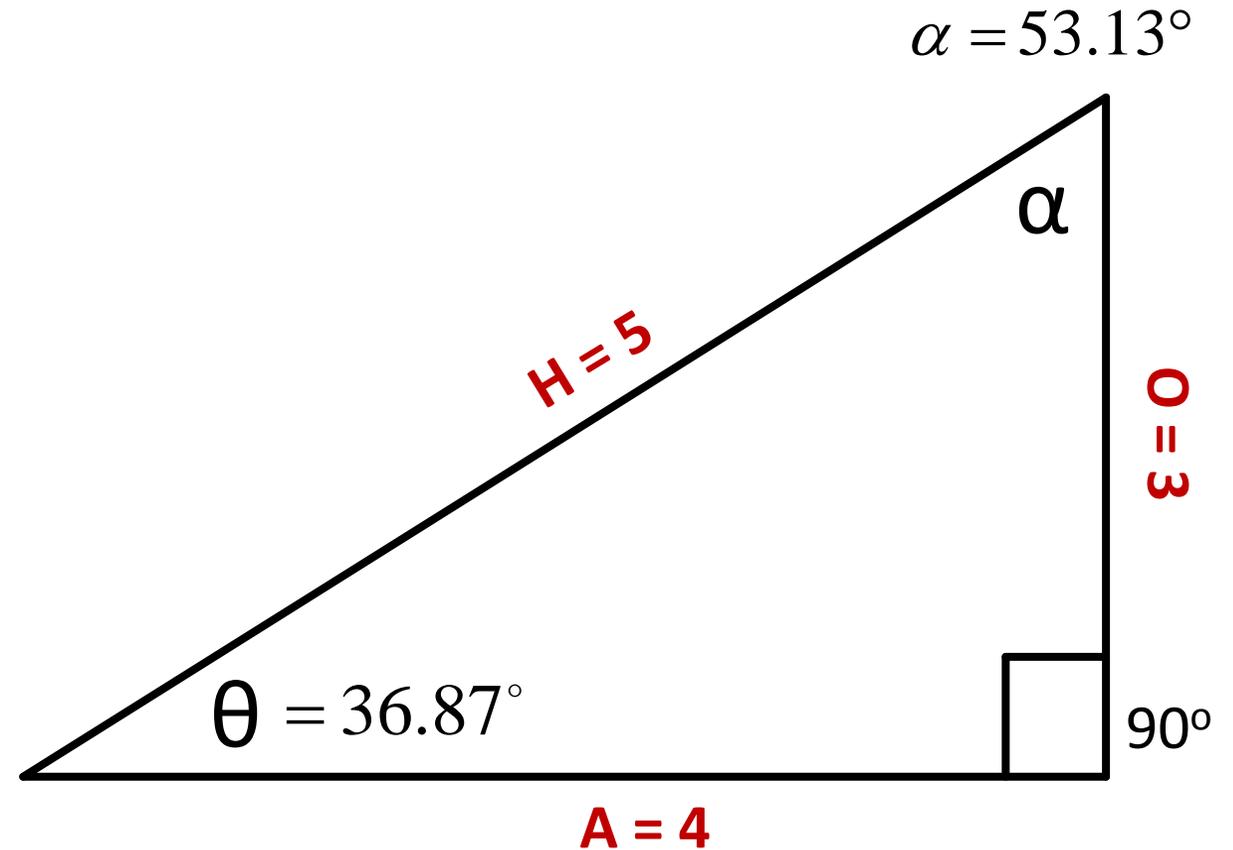
Answer Questions 1-3

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Question 1

Calculate the value of the hypotenuse and the angle θ in the following triangle
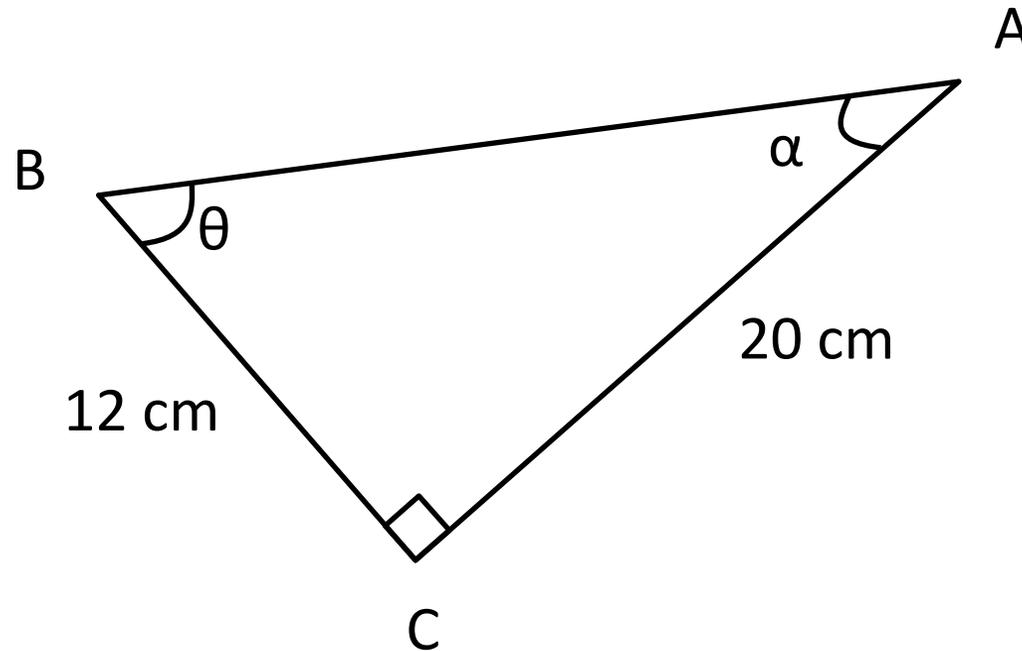
Question 1 - Answer

$$H = \sqrt{O^2 + A^2}$$

$$H = \sqrt{12^2 + 20^2}$$

$$H = \sqrt{144 + 400}$$

$$H = \sqrt{544} = 23.32cm$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{20cm}{12cm}$$

$$\tan\theta = 1.667$$

$$\theta = \tan^{-1}(1.667) = 59°$$

23.32cm

B

A

θ **= 59°**

α

20 cm

12 cm

C
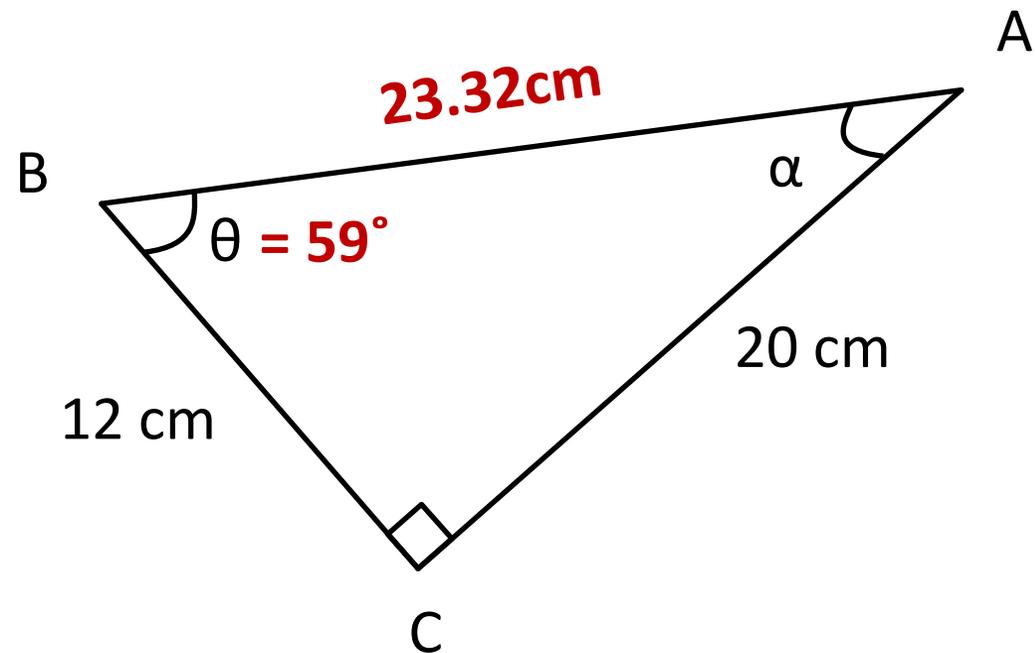
Question 2

Calculate the length of the side x, given that tan θ = 0.4
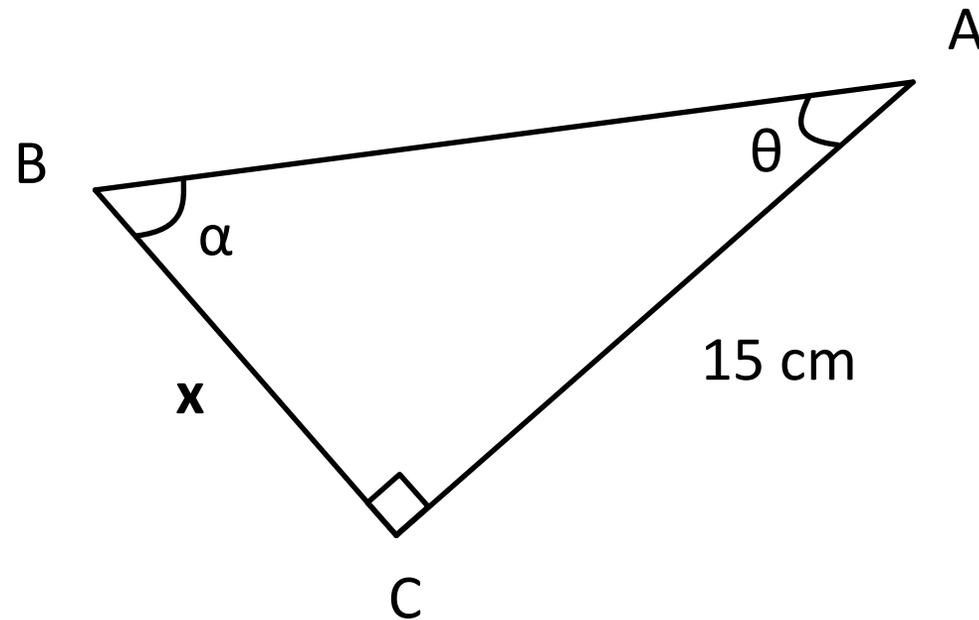
Question 2 - Answer

$$\tan \theta = \frac{opposite}{adjacent}$$

$$0.4 = \frac{x}{15}$$

$$x = (0.4)(15)$$

$$x = 6cm$$

tan θ = 0.4

A

θ

B

α

x = 6 cm

15 cm

C
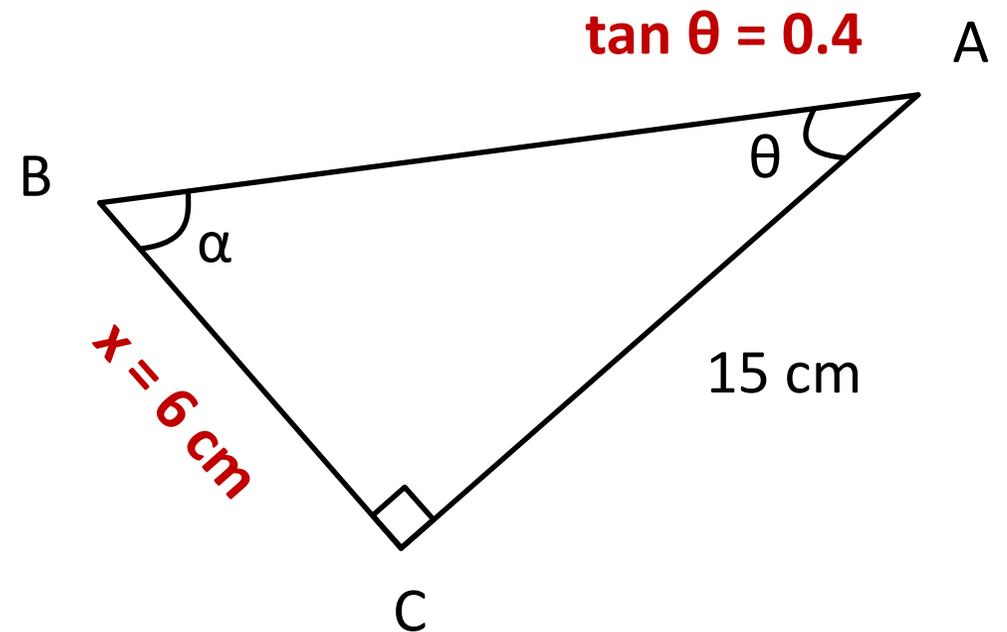
Question 3

Calculate the length of the side *x*, given that sin θ = 0.6
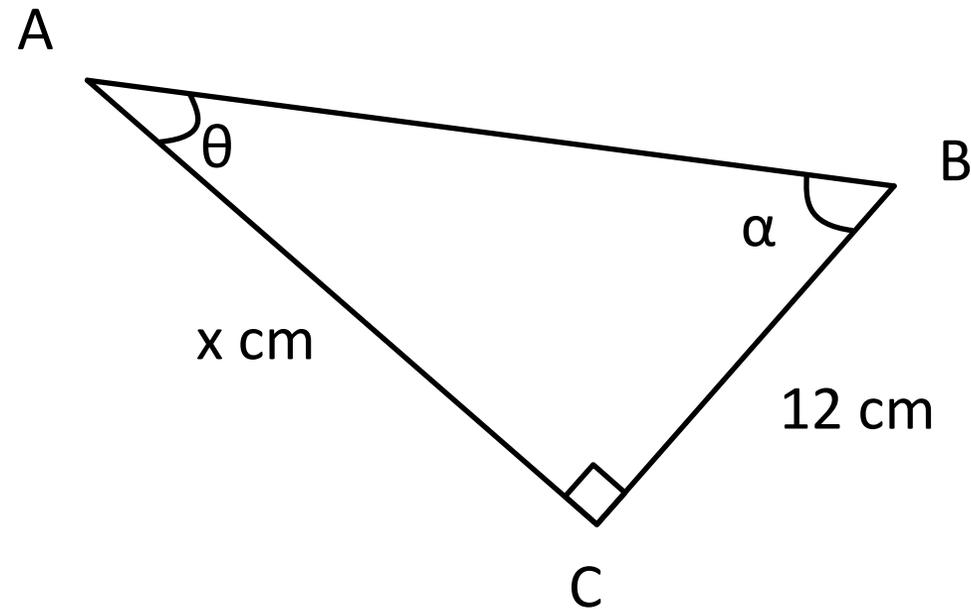
Question 3 - Answer

$$\sin \theta = \frac{opposite}{hypotenuse}$$

$$0.6 = \frac{12}{H}$$

$$H = \frac{12}{0.6}$$

$$H = 20cm$$

$$x = \sqrt{H^2 - A^2}$$

$$x = \sqrt{20^2 - 12^2}$$

$$x = \sqrt{400 - 144}$$

$$x = \sqrt{256} = 16cm$$

A

20 cm

sin θ =0.6

θ

B

α

x= 16cm

12 cm

C
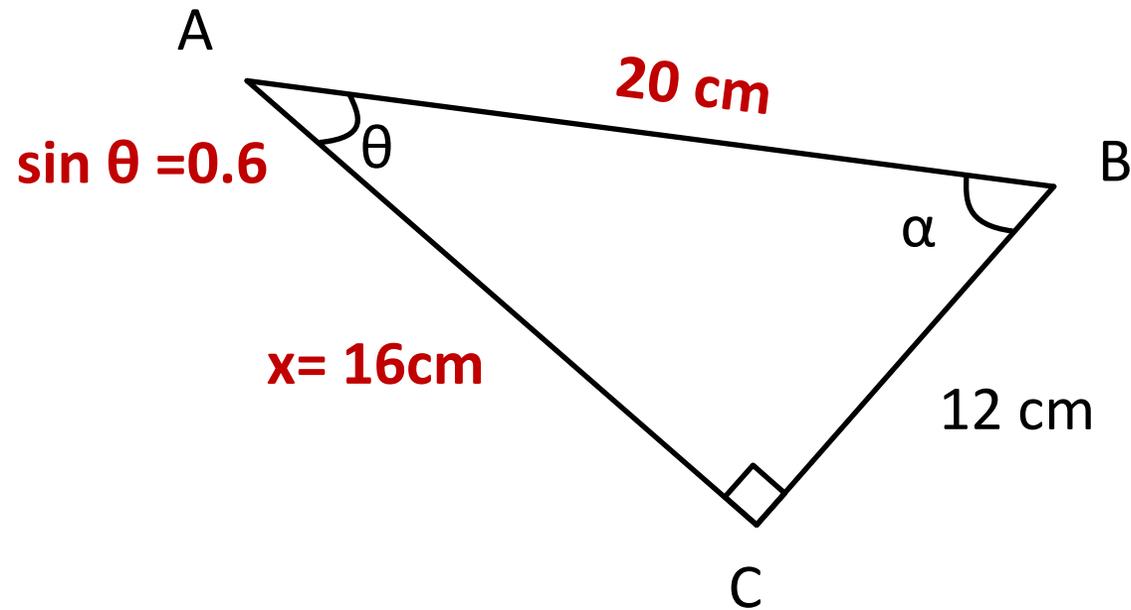
Per-Unit Quantities

Mathematics Review

- Ratios play an important part in estimating power system performance

  - Relationship between two quantities as a fraction

  - Used when the relationship of two pairs of values is the same, and one of two similarly related values is known

    - Example: if you can drive 120 miles in 2 hours, how many miles could you drive in 8 hours?

$$\frac{120\ miles}{2\ hours} = \frac{X\ miles}{8\ hours} = \frac{120\ miles(8\ \cancel{hours})}{2\ \cancel{hours}} = \frac{X\ miles\ (8\ \cancel{hours})}{8\ \cancel{hours}} = 480\ miles$$

Mathematics Review

- Quantities on the power system are often specified as a percentage or a per-unit of their base or nominal value

  – Makes it easier to see where a system value is in respect to its base value

  – How it compares between different parts of the system with different base values

  – Allow for a dispatcher to view the system and quickly obtain a feel for the voltage profile

Mathematics Review

- Assume that, at a certain substation, the voltage being measured is 510 kV on the 500 kV system. What is its per-unit value with respect to the nominal voltage?

Base or nominal voltage = 500 kV

Measured voltage = 510 kV

510 kV / 500 kV = <u>1.02 per-unit</u> or 102%

Mathematics Review


$$\frac{513.9 \text{ kV}}{500 \text{ kV}} = 1.03 \text{ per-unit}$$

$$\frac{226.98 \text{ kV}}{230 \text{ kV}} = 0.98 \text{ per-unit}$$

$$\frac{113.4 \text{ kV}}{115 \text{ kV}} = 0.98 \text{ per-unit}$$
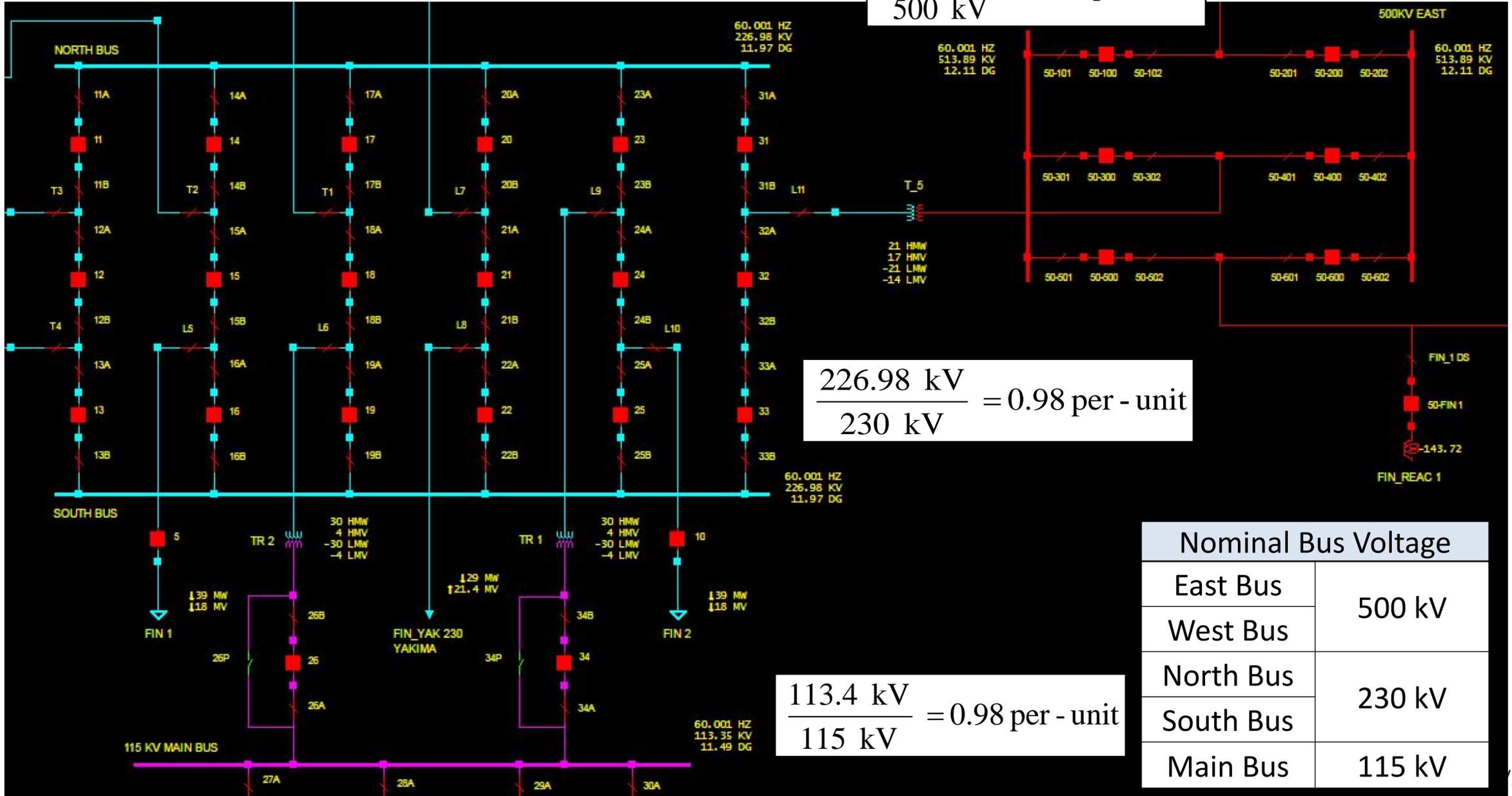
Nominal Bus Voltage	
East Bus	500 kV
West Bus	
North Bus	230 kV
South Bus	
Main Bus	115 kV

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Answer Questions 4 and 5

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Question 4

- Assume that the loss of a 1000 MW generating unit will typically result in a 0.2 Hz dip in system frequency

- Estimate the frequency dip for the loss of an 800 MW generating unit

Mathematics Review

$$\frac{0.2Hz}{1000MW} = \frac{(x)Hz}{800MW}$$

$$\frac{0.2Hz}{1000MW} = \frac{(x)Hz}{800MW}$$

$$\frac{(0.2Hz)(800MW)}{1000MW} = (x)Hz$$

$$x = \frac{160Hz}{1000} = .16Hz$$

Question 5

- Assume that the loss of a 500 MW generating unit will typically result in a 0.3 Hz dip in system frequency.

- Estimate the frequency dip for the loss of an 300 MW generating unit.

Mathematics Review

$$\frac{0.3 Hz}{500 MW} = \frac{(x) Hz}{300 MW}$$

$$\frac{0.3 Hz}{500 MW} = \frac{(x) Hz}{300 MW}$$

$$\frac{(300 MW)(0.3 Hz)}{500 MW} = (x) Hz$$

$$x = \frac{90 Hz}{500} = .18 Hz$$

Vectors

Vectors

- A vector is alternative way to represent a sinusoidal function with amplitude, and phase information

- Vectors are usually written in bold with an arrow over the top of the letter, ($\overrightarrow{\textbf{A}}$)

- A vectors length represents magnitude

- A vectors direction represents the phase angle

- Example: 10 miles east
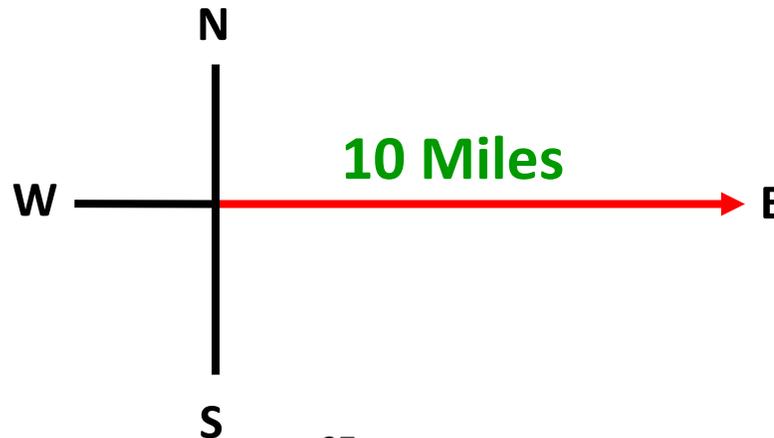
Vectors

- Horizontal line to the right is positive; horizontal line to the left is negative

- Vertical line going up is positive; vertical line going down is negative

- Arrowhead on the end away from the point of origin indicates the direction of the vector and is called the displacement vector

- Vectors can go in any direction in space

Vectors

- The difference between a scalar quantity and a vector:

    a) A scalar quantity is one that can be described with a single number, including any units, giving its size or magnitude

    b) A vector quantity is one that deals inherently with both magnitude and direction

Answer Questions 6 & 7

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Conceptual Question 6

- There are places where the temperature is +20° C at one time of the year and -20° C at another time.

- Do the plus and minus signs that signify positive and negative temperatures imply that temperature is a vector quantity?

Question 7

Which of the following statements, if any, involves a vector?

  a) I walked two miles along the beach

  b) I walked two miles due north along the beach

  c) A ball fell off a cliff and hit the water traveling at 17 miles per hour

  d) A ball fell off a cliff and traveled straight down 200 feet

  e) My bank account shows a negative balance of -25 dollars

Vectors

- When adding vectors, the process must take into account both the magnitude and direction of the vectors

- When adding two vectors, there is always a resultant vector, R, and the addition is written as follows:

$$\vec{R} = \vec{A} + \vec{B}$$

Vectors

Example: Adding vectors in the same direction

- Vector A has a length of 2 and a direction of 90°

- Vector B has a length of 3 and a direction of 90°

Vectors

Example: Adding vectors in the opposite direction

- Vector $\vec{A}$ has a length of 2 and a direction of 90°

- Vector $\vec{B}$ has a length of 3 and a direction of 270^0

$\vec{A}$

$\vec{B}$

90°

180°

0°

$\vec{R}$

270°
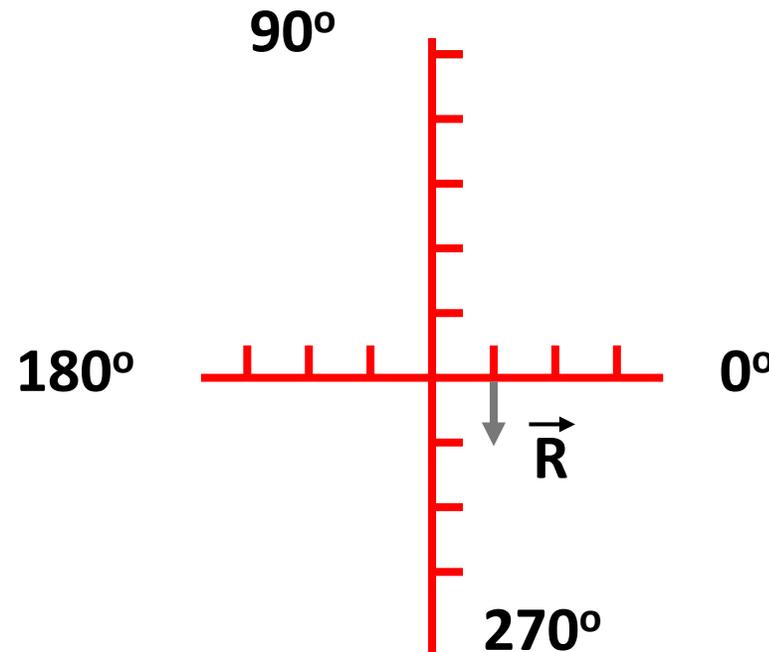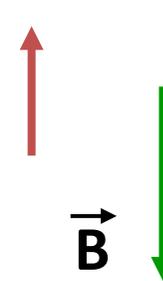
Answer Question 8

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Question 8

Two vectors, $\vec{A}$ and $\vec{B}$, are added to give a resultant vector, $\vec{R}$. The magnitudes are 3 and 8 meters, respectively, but the vectors can have any orientation. What is the maximum and minimum possible values for the magnitude of $\vec{R}$?

Maximum: 3+8=11

Minimum: (-3)+(-8)=(-11)

90º

$\vec{A}$

180º   0º

$\vec{B}$

270º
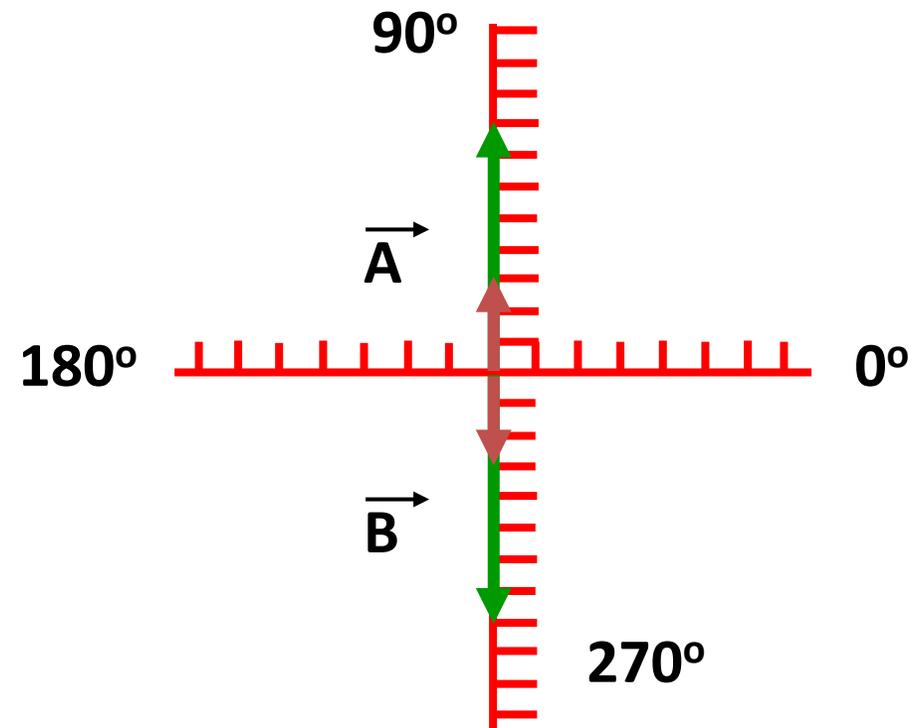
Vectors

- Subtraction of one vector from another is carried out in a way that depends on the following:

  When a vector is multiplied by -1, the magnitude of the vector remains the same, but the direction of the vector is reversed

- Vector subtraction is carried out exactly like vector addition except that one of the vectors added is multiplied by the scalar factor of -1

Vectors
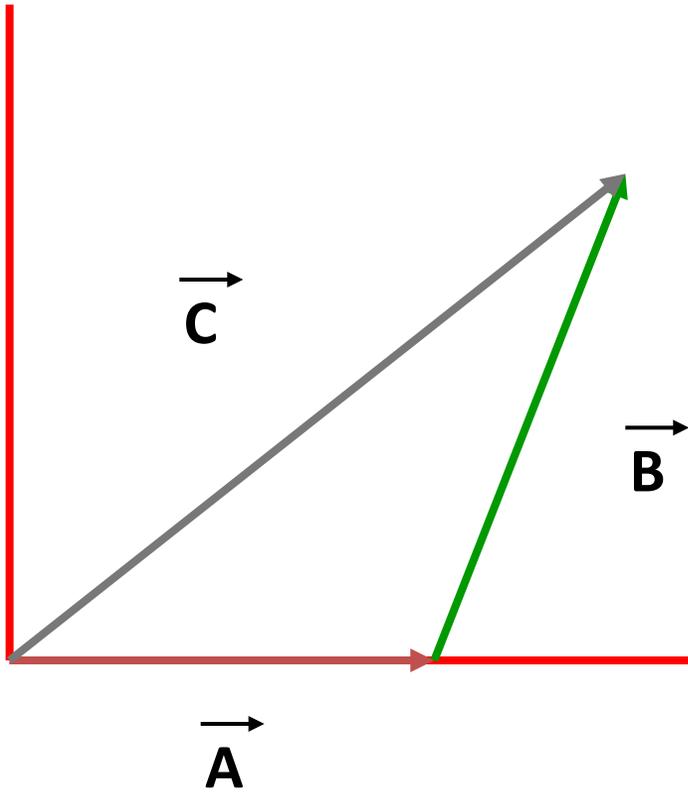
$$\overrightarrow{C} = \overrightarrow{A} + \overrightarrow{B}$$
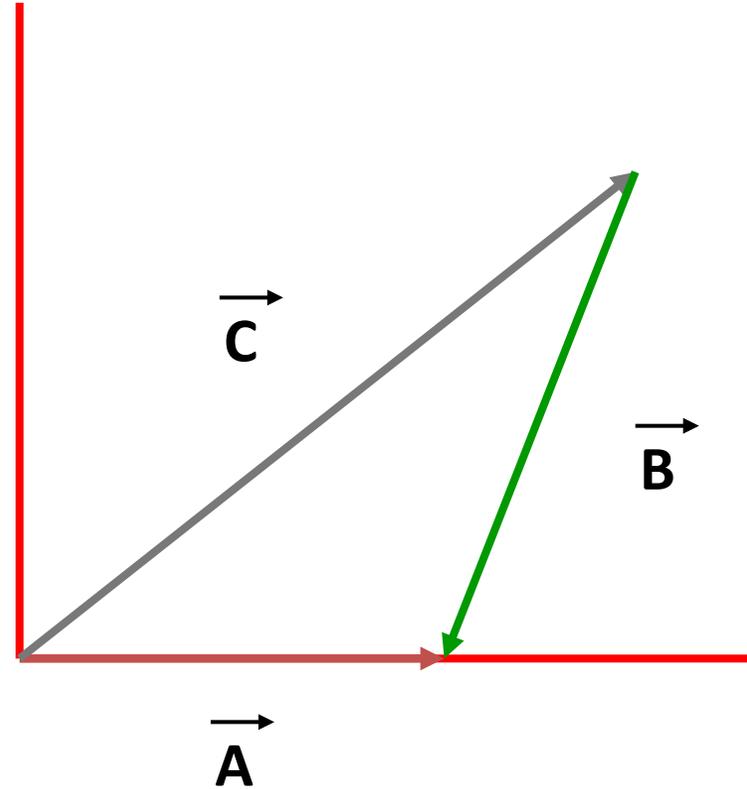
$$\overrightarrow{A} = \overrightarrow{C} + (-\overrightarrow{B})$$

Vectors

- If the magnitude and direction of a vector is known, it is possible to find the components of the vector

- The process is called "resolving the vector into its components"

- If the vector components are perpendicular and form a right triangle, the process can be carried out with the aid of trigonometry

Vectors

- To calculate the sum of two or more vectors using their components (x and y) in the vertical and horizontal directions, trigonometry is used
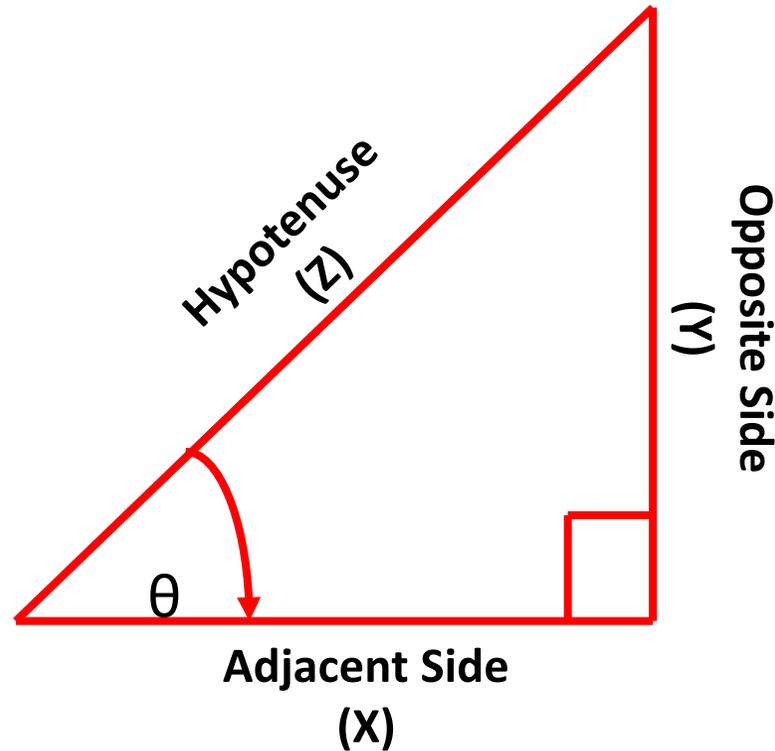
Vectors

- The Pythagorean Theorem is a special relationship that exists in any triangle and describes the relationship between the lengths of the sides of a right triangle

$$z^2 = x^2 + y^2$$

- Three basic trigonometric functions defined by a right triangle are:

$\sin \theta = $ y/z $ = $ opposite side/hypotenuse

$\cos \theta = $ x/z $ = $ adjacent side/hypotenuse

$\tan \theta = $ y/x $ = $ opposite side/adjacent side

$\tan \theta = \sin \theta / \cos \theta$

Vectors

- To find theta, the inverse of the trigonometric function must be used

$$\theta = \sin^{-1} y/z$$

$$\theta = \cos^{-1} x/z$$

$$\theta = \tan^{-1} y/x$$

- When adding vectors, magnitude is found by:

$$\vec{R} = \sqrt{(\vec{R_x})^2 + (\vec{R_y})^2}$$

- The direction of the resultant, R, is found by:

$$\theta = \sin^{-1} (R_y/R)$$

Vectors

- Vectors can be added either in the same direction or in opposite directions

54
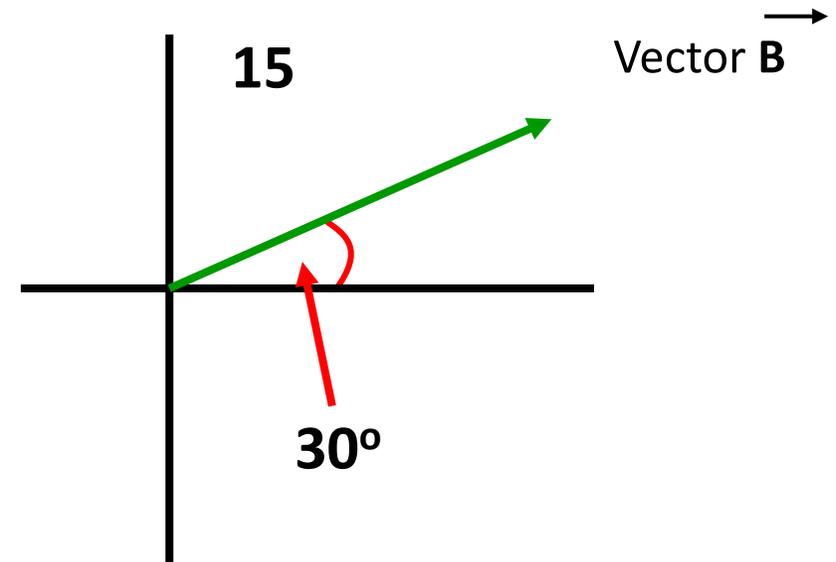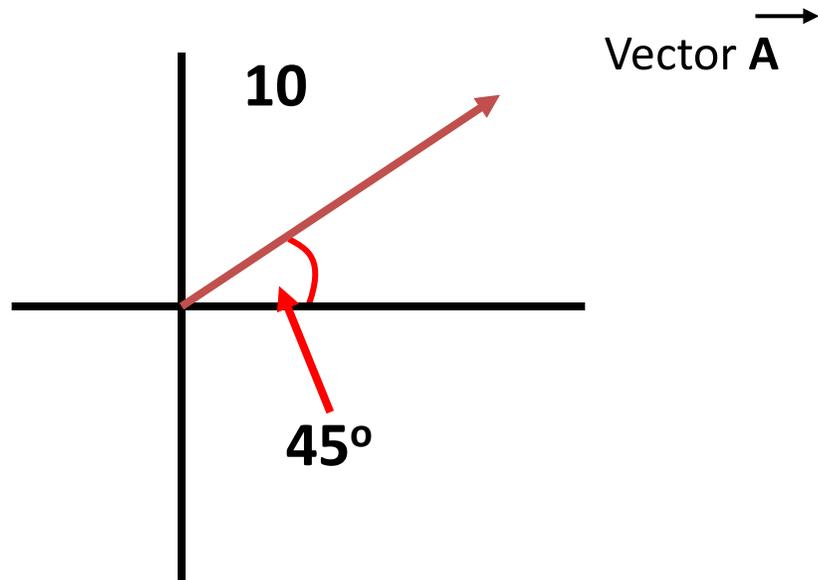
Vectors

Adding vectors:

Vectors

$\text{Vector } \overrightarrow{B}$
15

30º

$\text{Vector } \overrightarrow{A}$
10

A_y

45º

A_x

Vector $A_x = (10)(\cos 45º) = 7.07$
Vector $A_y = (10)(\sin 45º) = 7.07$

Vectors

Vector $B_x = (15)(\cos 30º) = 12.99$

Vector $B_y = (15)(\sin 30º) = 7.5$
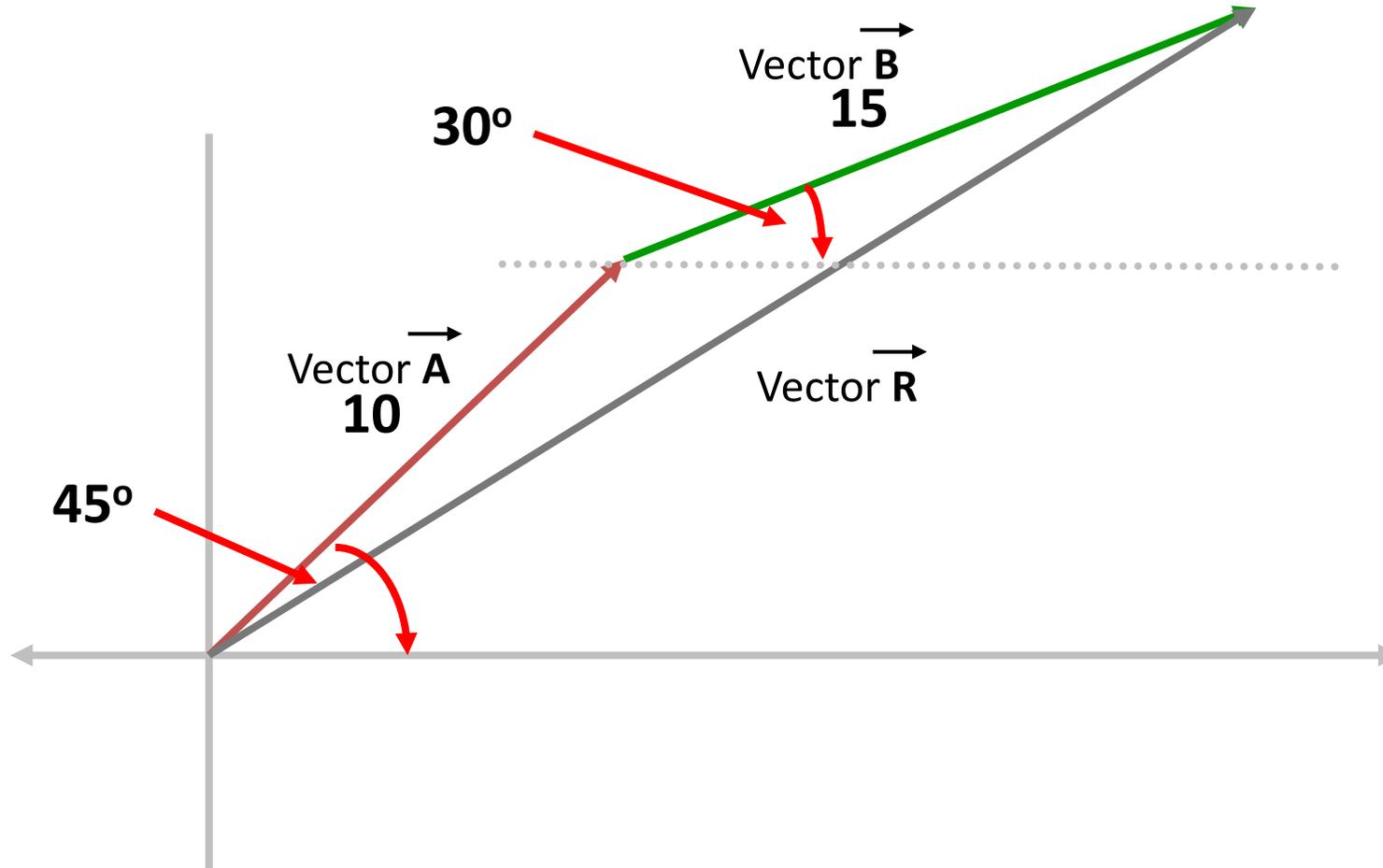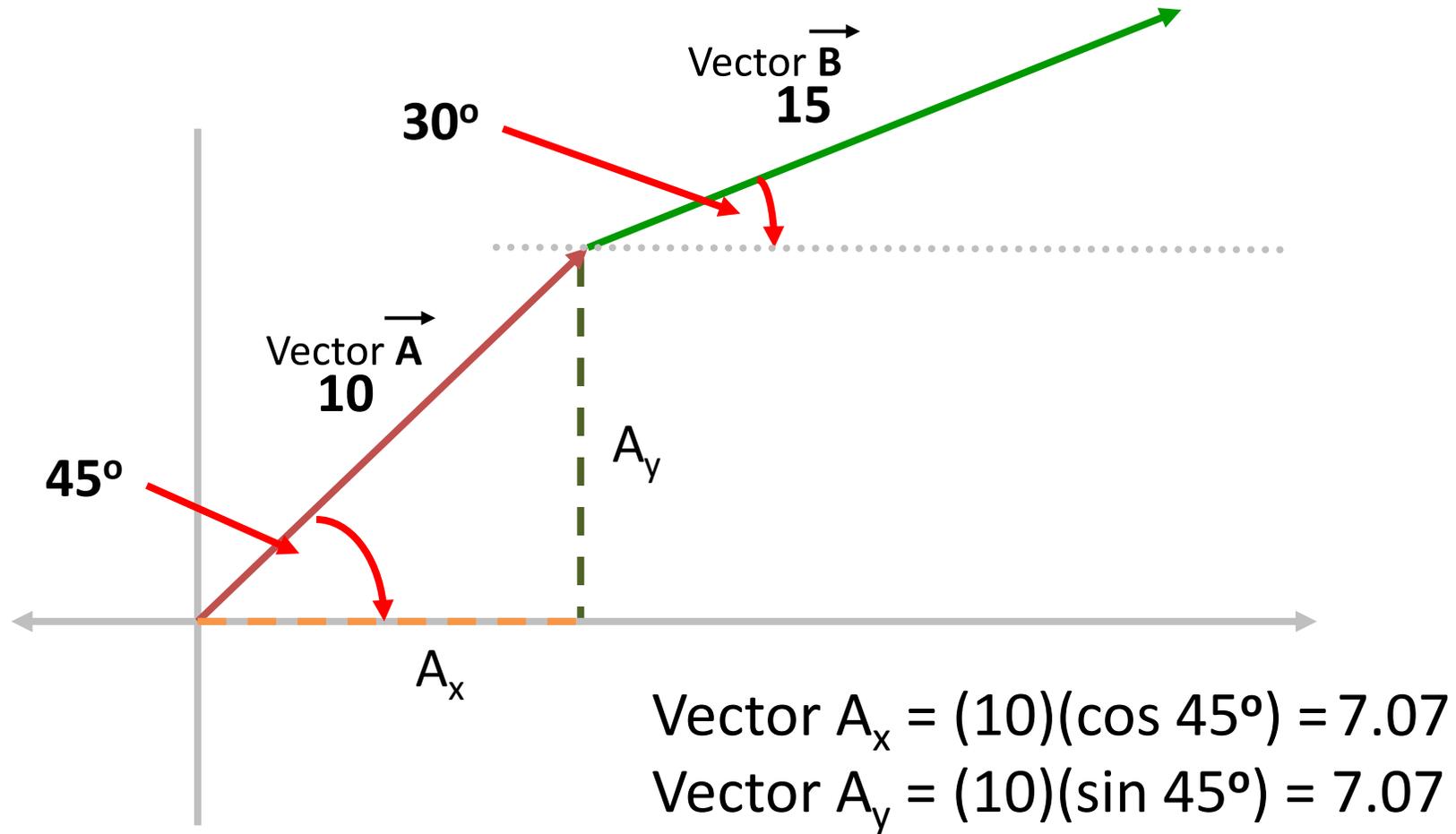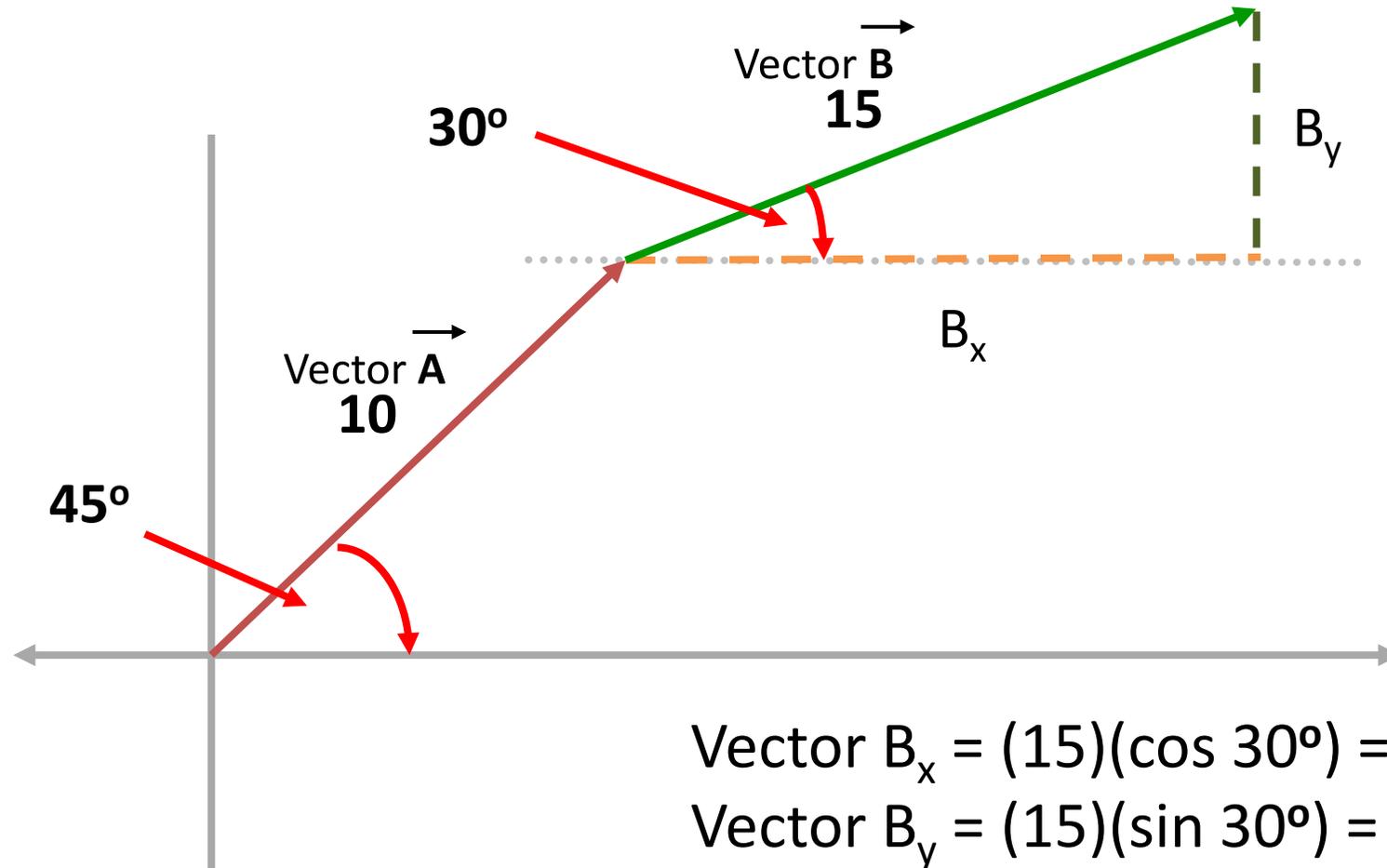
Vectors

Determine the resulting vectors, R_x and R_y:

$$\vec{R_x} = \vec{A_x} + \vec{B_x}$$

$\vec{R_x} = 7.07 + 12.99$

$\vec{R_x} = 20.06$

$$\vec{R_y} = \vec{A_y} + \vec{B_y}$$

$\vec{R_y} = 7.07 + 7.5$

$\vec{R_y} = 14.57$

30º

Vector $\vec{B}$

Vector $\vec{A}$

$\vec{R_y} = 14.57$

$\vec{R_x} = 20.06$

45º
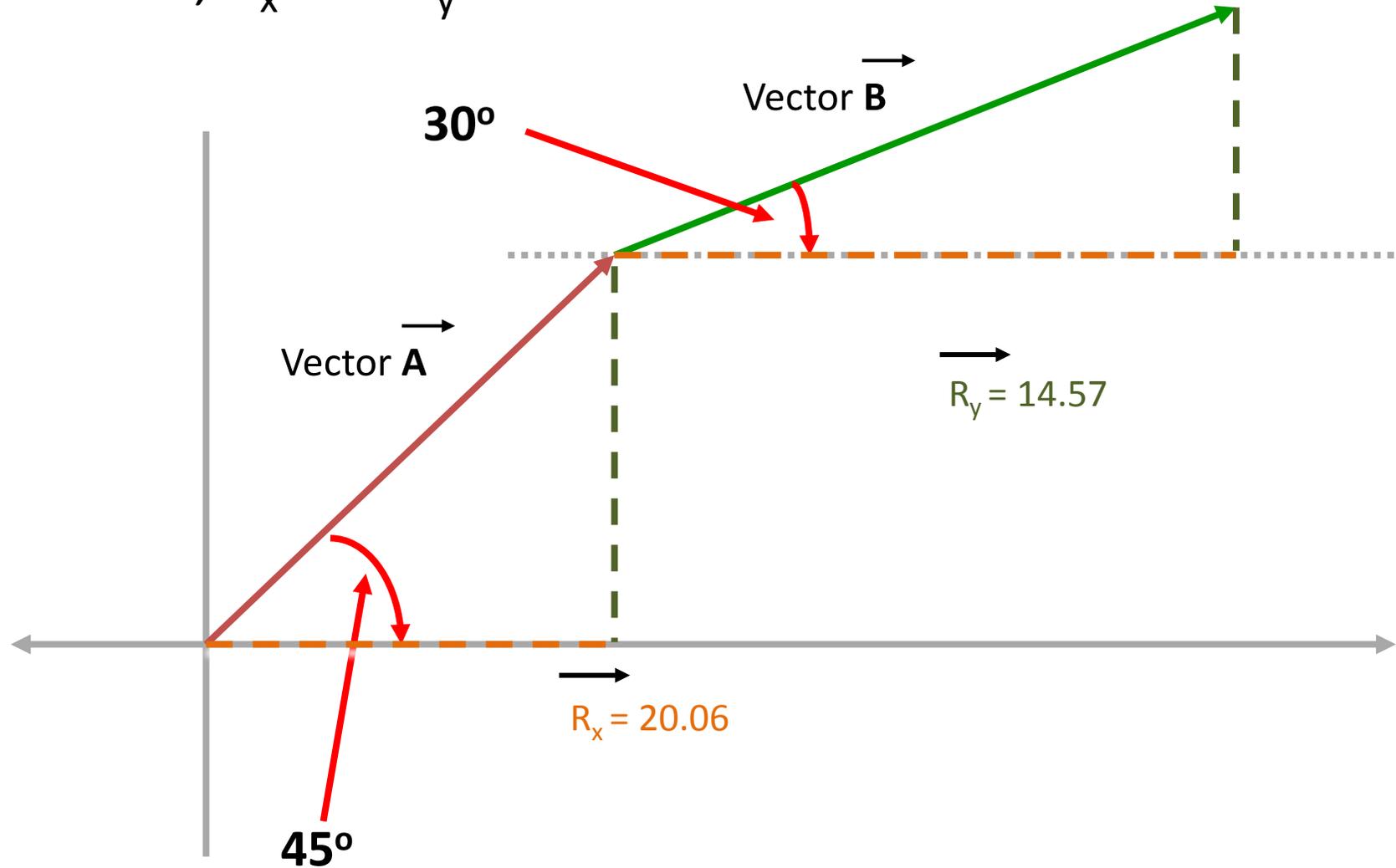
Vectors

$$\vec{R} = \sqrt{\vec{R}_x^2 + \vec{R}_y^2}$$

$$\vec{R} = \sqrt{20.06^2 + 14.57^2}$$

$$\vec{R} = \sqrt{614.7}$$

$$\vec{R} = 24.79$$

$$\theta = \sin^{-1}\left(\frac{R_y}{R}\right)$$

$$\theta = \sin^{-1}\left(\frac{14.57}{24.79}\right)$$

$$\theta = \sin^{-1}(.587)$$

$$\theta = 35.9°$$

Vector **B**

Vector **A**

Vector **R**
24.79

14.57

20.06

35.9º
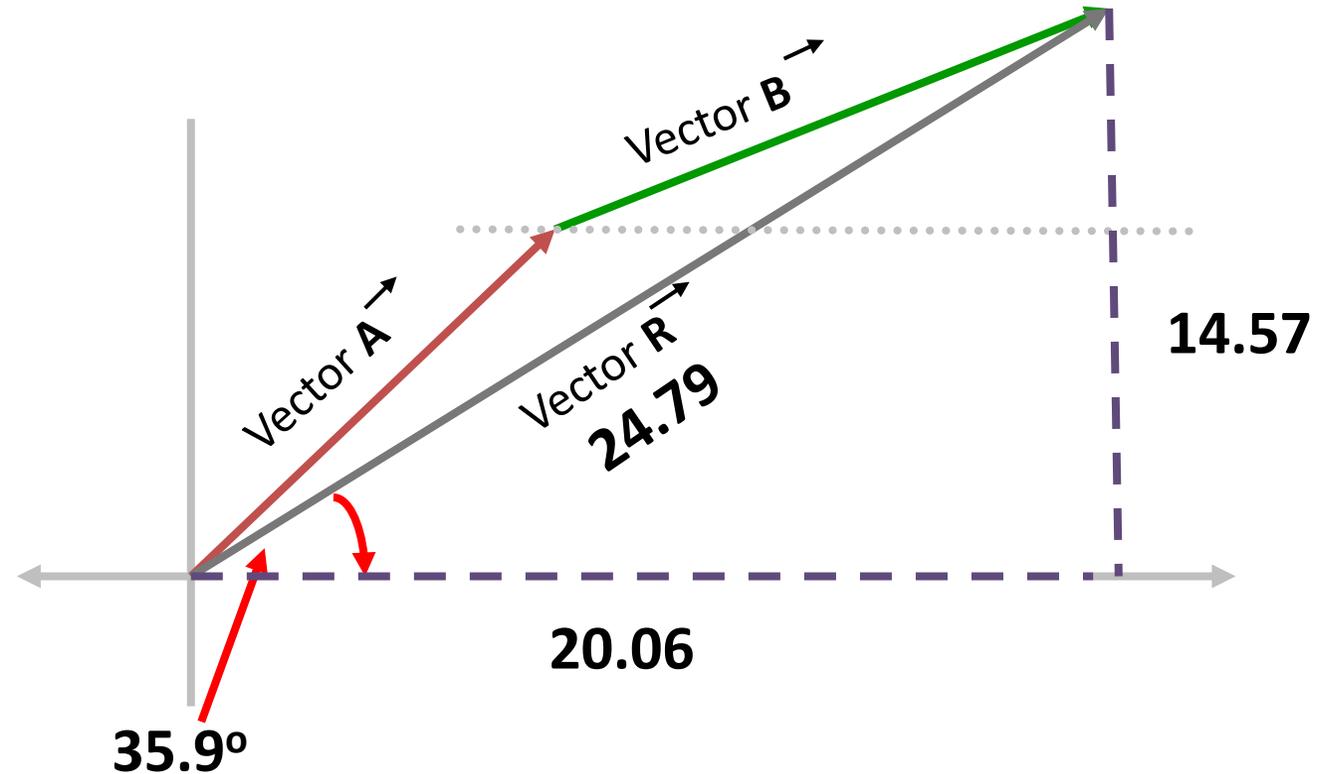
Vectors

- Polar notation expresses a vector in terms of both a magnitude and a direction, such as:

where:

M ∠ °

M is the magnitude of the vector
° is the direction in degrees

Example:
Vector with a magnitude of 10 and a direction of -40 degrees 10 ∠ -40°

Vectors

- Multiplication in polar notation:

    Multiply the magnitudes/add the angles

    $(50 \angle 25^o)(25 \angle 30^o) = 1250 \angle 55^o$

- Division in polar notation:

    Divide the magnitudes/subtract the angles

$$\frac{(50 \angle 25^o)}{(25 \angle 30^o)} = 2 \angle \text{-}5^o$$

Answer Question 9

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Question 9

A displacement vector $\vec{R}$ has a magnitude of $\vec{R}$ = 175 m and points at an angle of 50º relative to the x-axis. Find the x and y components of this vector?
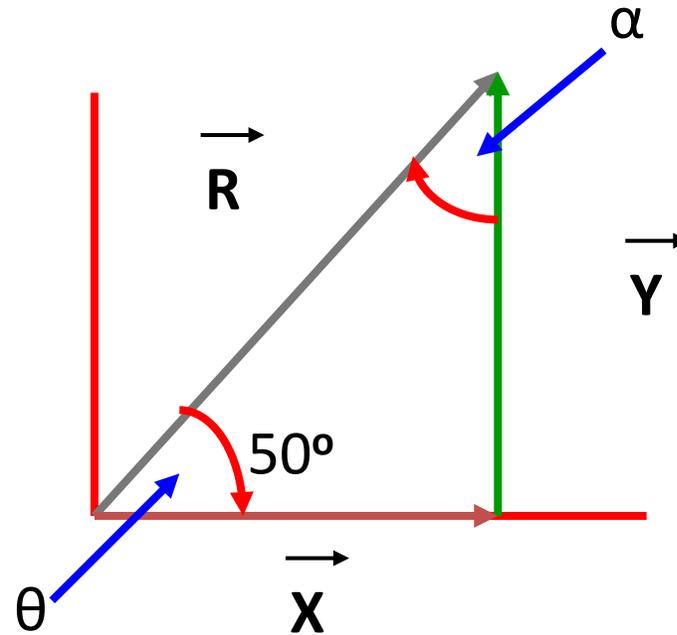
Question 9

$$\sin\theta = \frac{O}{H} = \frac{Y}{R}$$

$$\sin 50° = \frac{Y}{175m}$$

$$Y = (175m)\sin 50° = 134.06m$$

$$X = \sqrt{H^2 - O^2} = \sqrt{R^2 - Y^2}$$

$$X = \sqrt{175m^2 - 134.06m^2} = \sqrt{12652.92m^2} = 112.5m$$
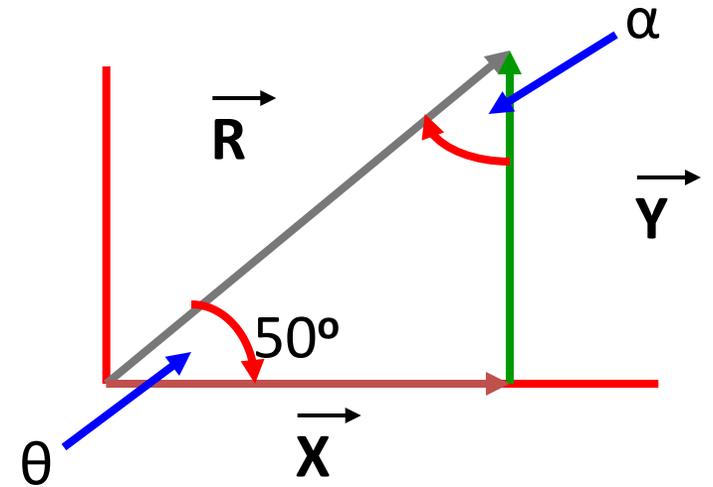
Summary

- Discussed the different components of Right Triangles

- Reviewed the basics of Trigonometry

- Computed different Per-Unit Quantities

- Characterized the two components of Vectors

Questions?

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