Basic Electrical Theory

Mathematics Review

PJM State & Member Training Dept.

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Objectives

By the end of this presentation the Learner should be able to:

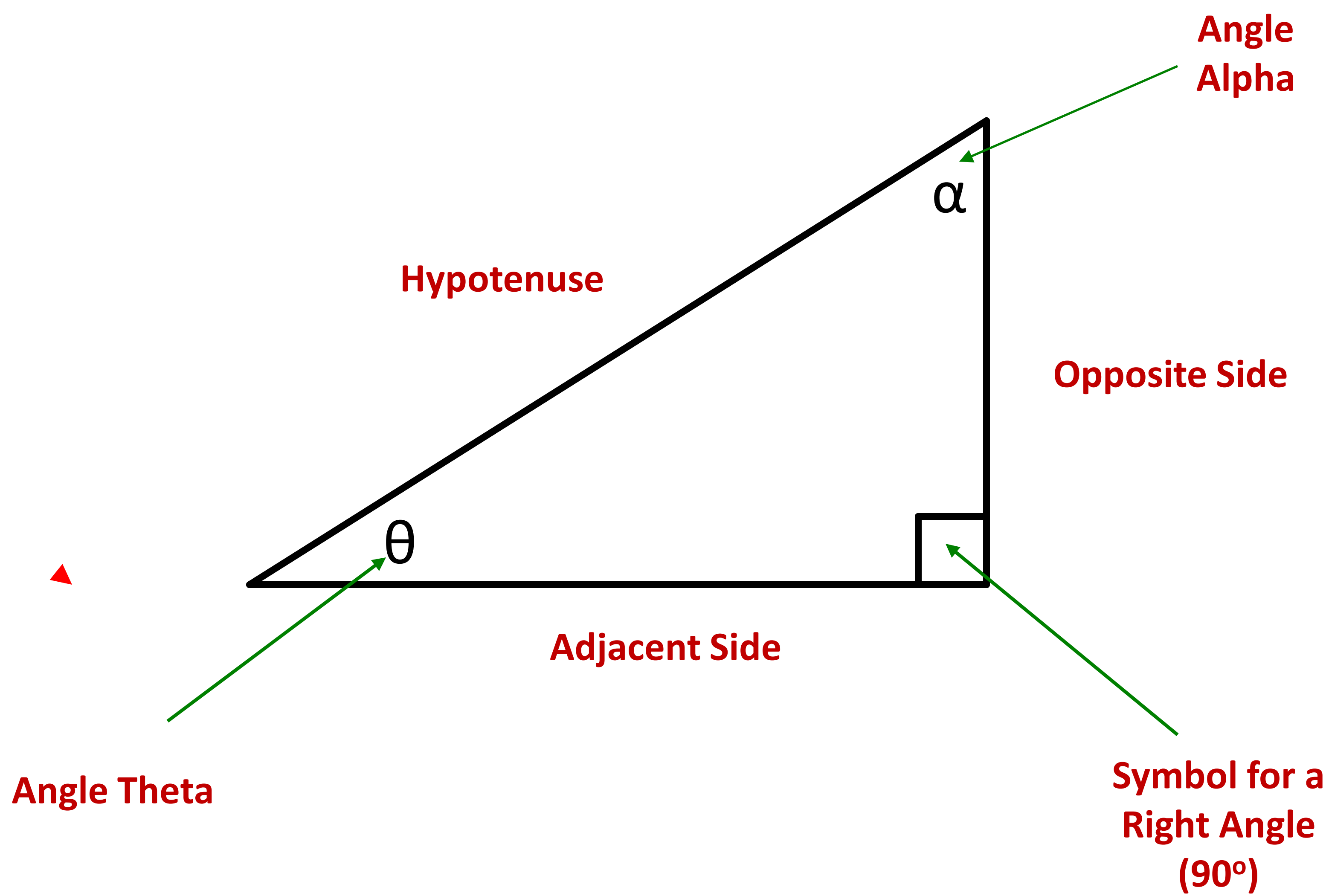
- Use the basics of trigonometry to calculate the different components of a right triangle
- Compute Per-Unit Quantities
- Identify the two components of Vectors

Right Triangles

Mathematics Review

- To be able to understand basic AC power concepts, a familiarization with the relationships between the angles and sides of a right triangle is essential
- A right triangle is defined as a triangle in which one of the three angles is a right angle always equal to 90^o
- Two of the sides which form the right triangle are designated as the adjacent and opposite sides with respect to the angle “theta”
- The third side of the right triangle is the longest side and is called the hypotenuse

Mathematics Review



- Given the lengths of two sides of a right triangle, the third side can be determined using the Pythagorean Theorem
- The square of the hypotenuse is equal to the sum of the squares of the remaining two sides:

$$\text{Hypotenuse}^2 = \text{Opposite}^2 + \text{Adjacent}^2$$

Mathematics Review

- Given a right triangle whose hypotenuse is 10, and the adjacent side is 6, what is the length of the opposite side?

$$\text{Hypotenuse}^2 = \text{Adjacent}^2 + \text{Opposite}^2$$

$$10^2 = 6^2 + \text{Opposite}^2$$

$$100 = 36 + \text{Opposite}^2$$

$$100 - 36 = \text{Opposite}^2$$

$$\sqrt{100 - 36} = \text{Opposite}$$

$$\sqrt{64} = \text{Opposite}$$

$$\text{Opposite} = 8$$

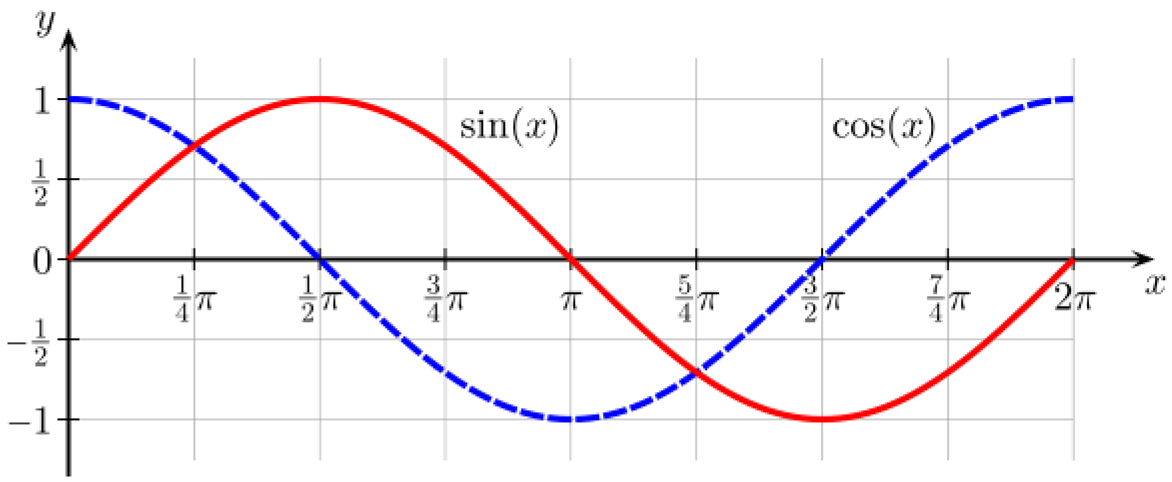
Mathematics Review

- Once the sides are known, the next step in solving the right triangle is to determine the two unknown angles of the right triangle
- Three angles of any triangle always add up to 180^o
- In solving a right triangle, the remaining two unknown angles must add up to 90^o
- Basic trigonometric functions are needed to solve for the values of the unknown angles

Trigonometry

Mathematics Review

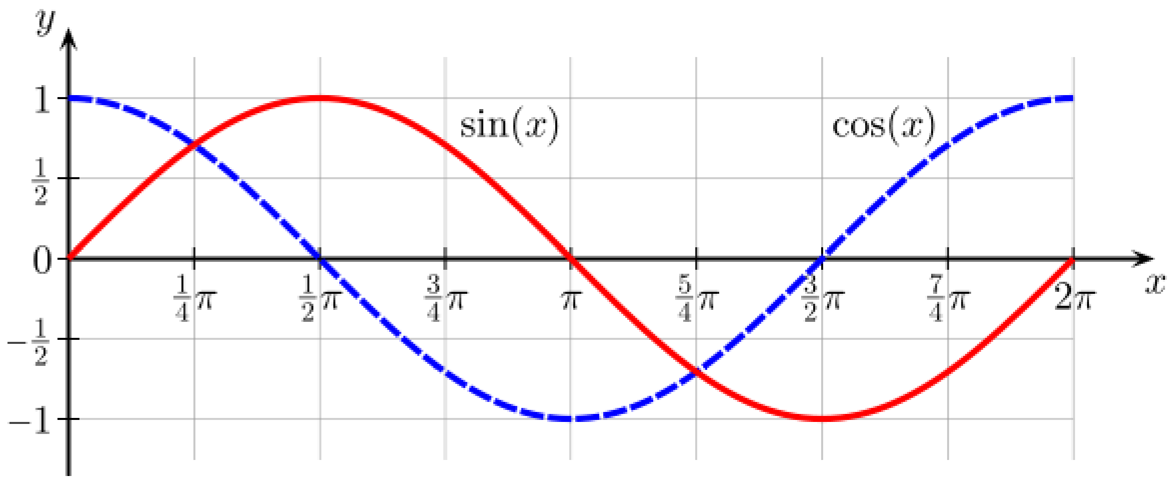
- The sine function is a periodic function in that it continually repeats itself



- In order to solve right triangles, it is necessary to know the value of the sine function between 0^o and 90^o
- Sine of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the hypotenuse

SIN θ = Opposite side / Hypotenuse

- Cosine function is a periodic function that is identical to the sine function except that it leads the sine function by 90^{o}



- As an example, the cosine function at 0^o is 1 whereas the sine function does not reach the value of 1 until 90^o
- Cosine function of either of the unknown angles of a right triangle is the ratio of the length of the adjacent side to the length of the hypotenuse

COS θ = Adjacent side / Hypotenuse

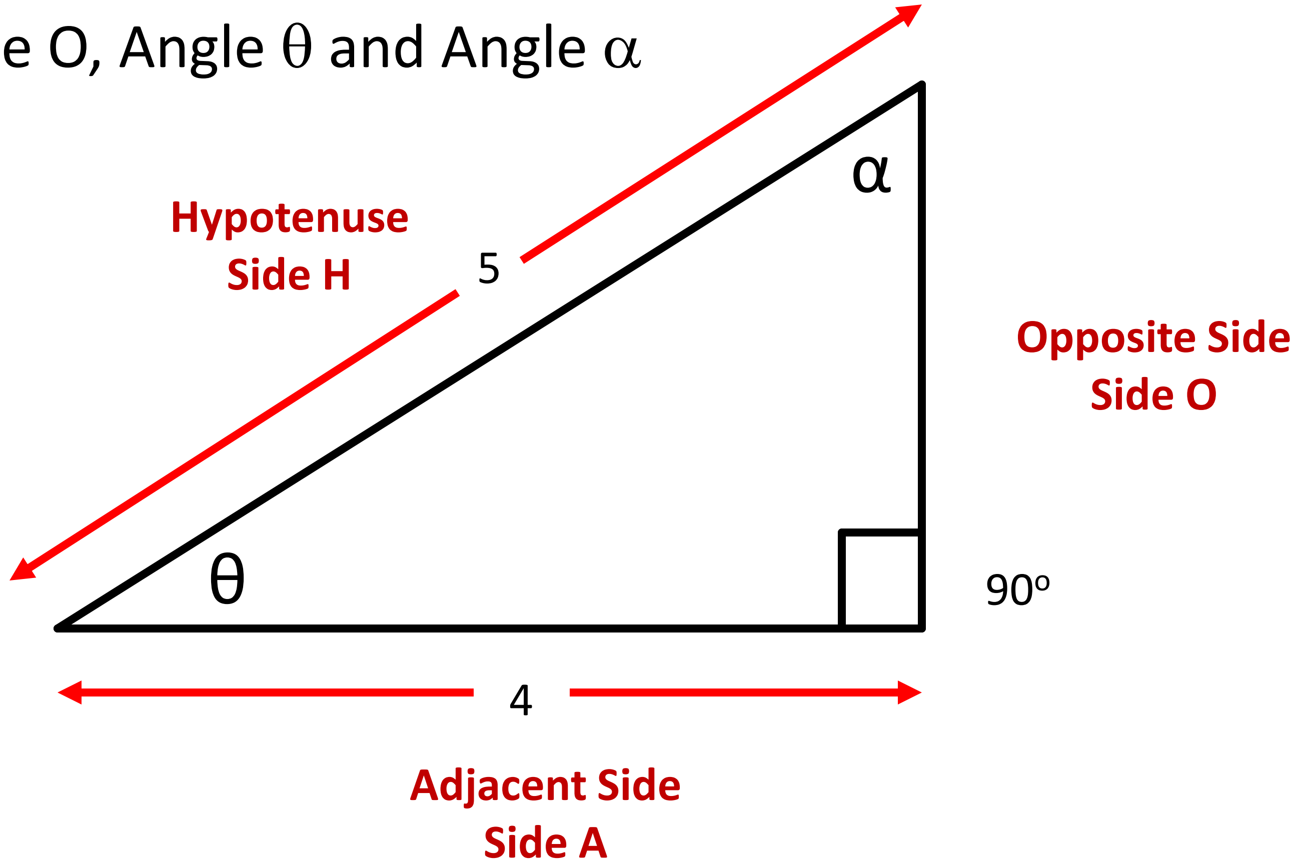
Mathematics Review

- The tangent function of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the adjacent side

TAN θ = Opposite side / Adjacent side

Mathematics Review

- Example:
- Given: Side H = 5, Side A = 4
- Find: Side O, Angle θ and Angle α



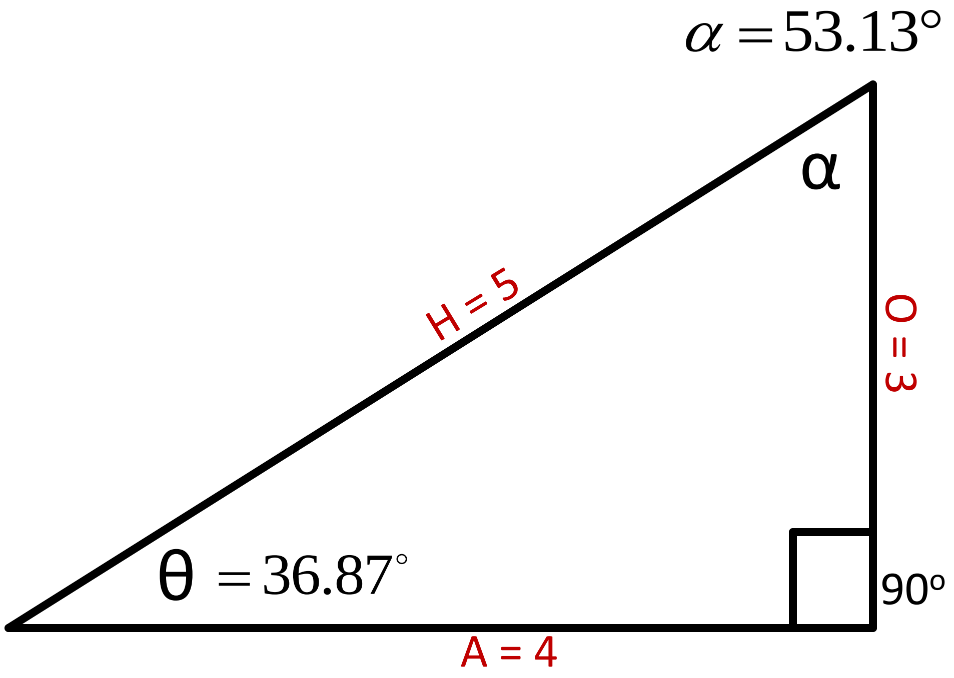
Mathematics Review

- Find Side O: $$H^2 = A^2 + O^2$$ $$25 = 16 + O^2$$ $$25 - 16 = O^2$$ $$\sqrt{25-16} = O$$ $$O = \sqrt{9} = 3$$

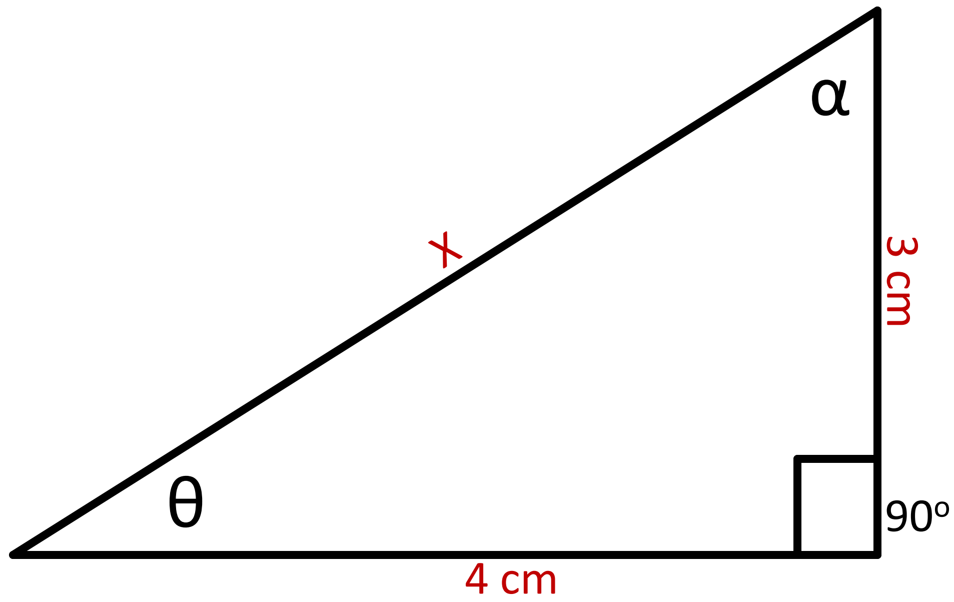
- Find θ: $\sin\theta = \dfrac{O}{H} = \dfrac{3}{5} = .6$

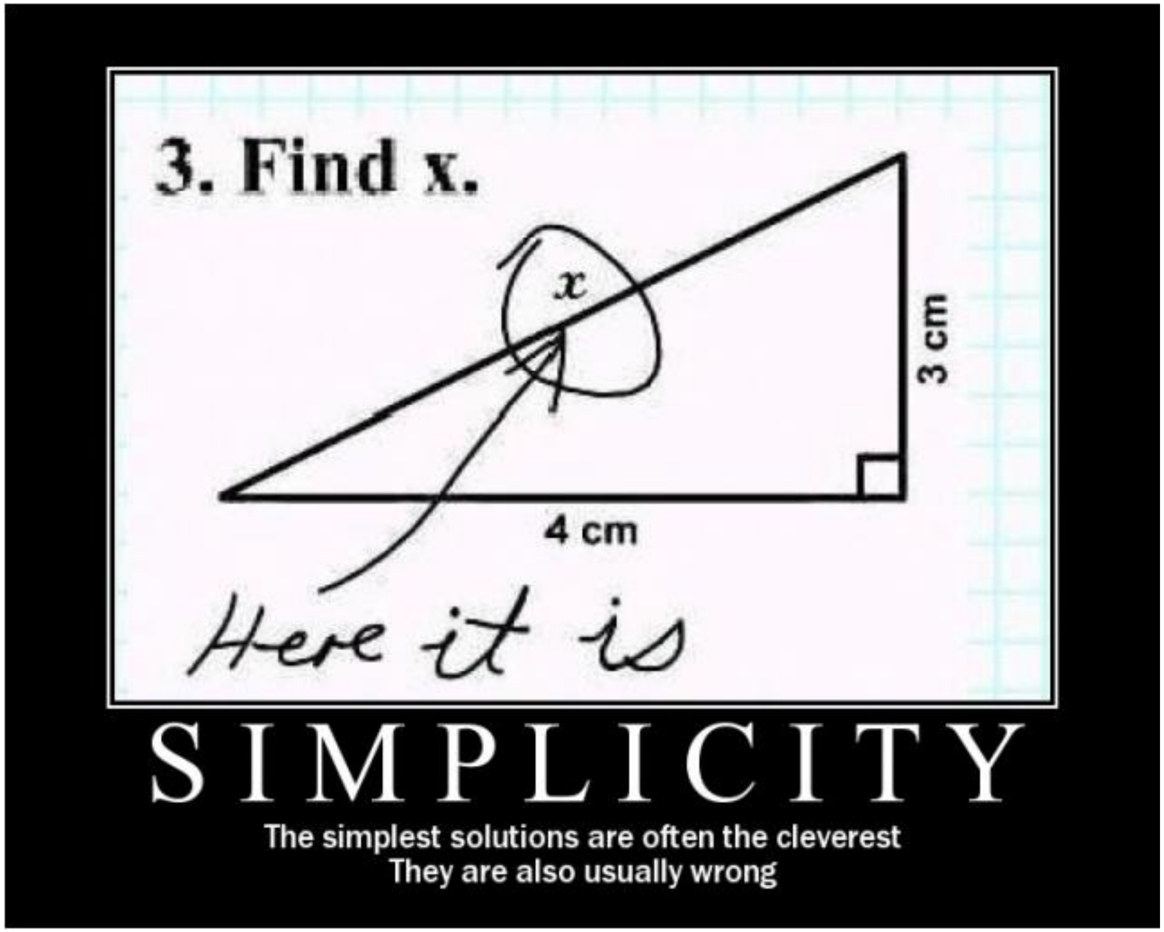
  $$\sin(.6)^{-1} = 36.87^\circ$$

- Find α: $180^\circ - 90^\circ - 36.87^\circ = 53.13^\circ$



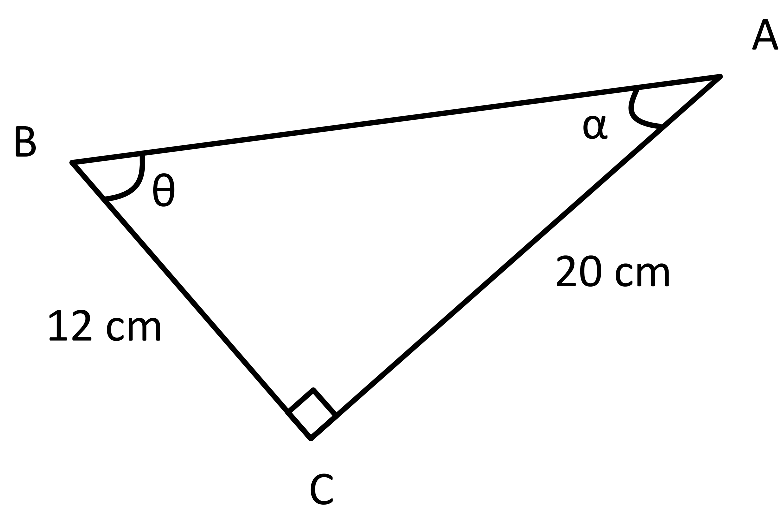
Find X




3. Find x.
x
3 cm
4 cm
Here it is
SIMPLICITY
The simplest solutions are often the cleverest
They are also usually wrong

Question 1

Calculate the value of the hypotenuse and the angle θ in the following triangle



Question 1 - Answer

$$H = \sqrt{O^2 + A^2}$$

$$H = \sqrt{12^2 + 20^2}$$

$$H = \sqrt{144 + 400}$$

$$H = \sqrt{544} = 23.32cm$$

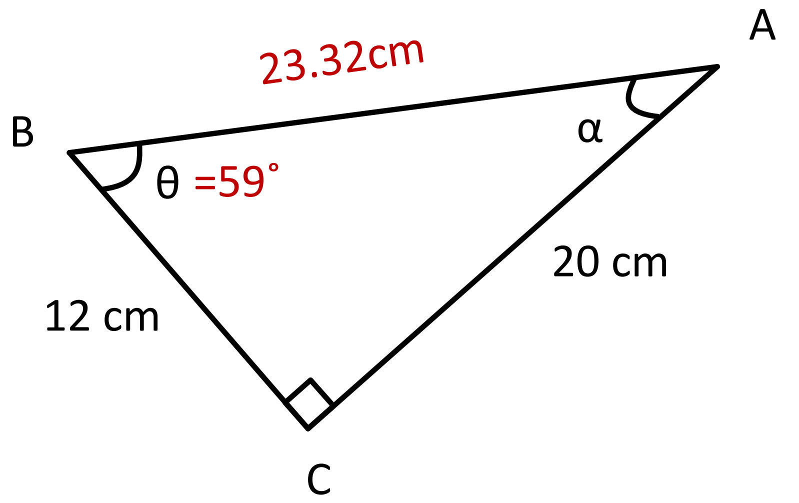
$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\cos\theta = \frac{A}{H}$$

$$\cos\theta = \frac{12cm}{23.32cm}$$

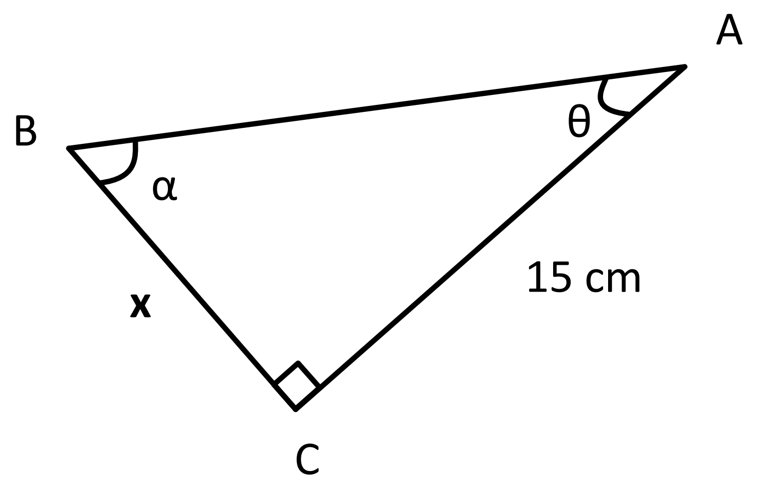
$$\cos\theta = .514$$

$$\theta = \cos(.514)^{-1} = 59^\circ$$



Question 2

Calculate the length of the side x, given that tan θ = 0.4



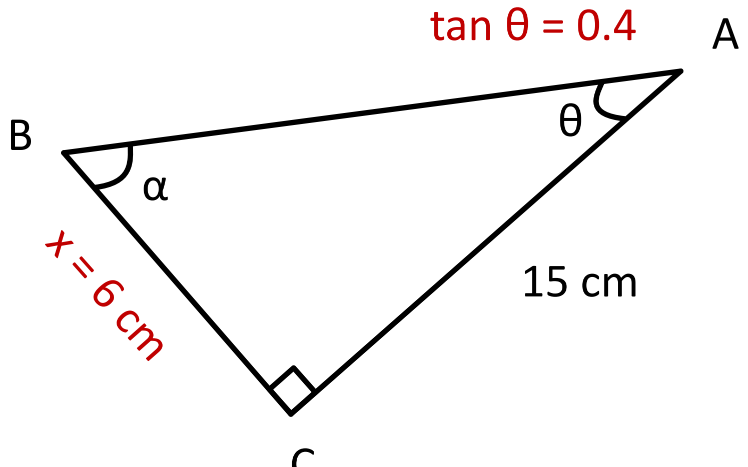
Question 2 - Answer

$$\tan\theta = \frac{opposite}{adjacent}$$

$$0.4 = \frac{x}{15}$$

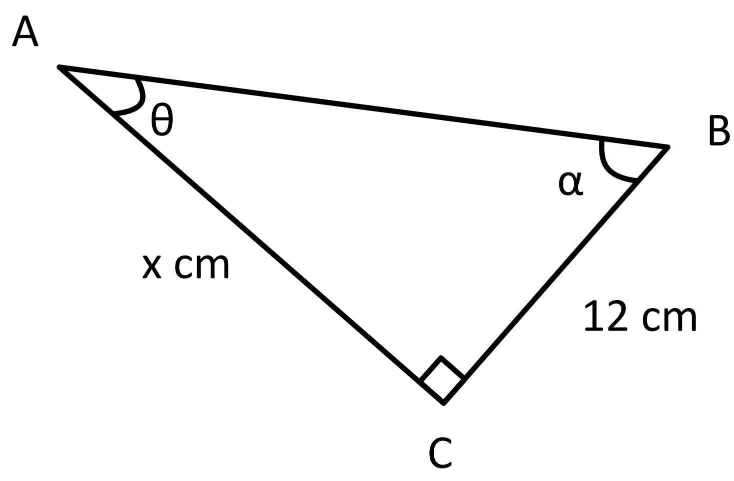
$$x = (0.4)(15)$$

$$x = 6cm$$



Question 3

Calculate the length of the side *x*, given that sin θ = 0.6



Question 3 - Answer

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$0.6 = \frac{12}{H}$$

$$H = \frac{12}{0.6}$$

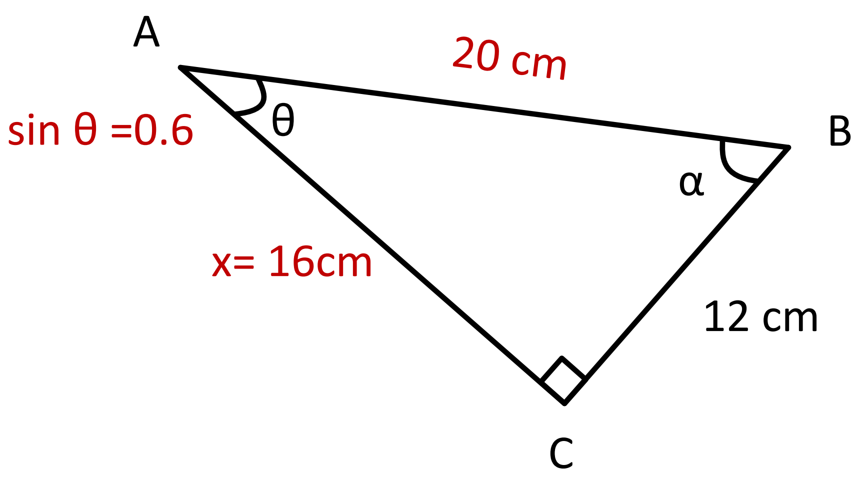
$$H = 20cm$$

$$x = \sqrt{H^2 - A^2}$$

$$x = \sqrt{20^2 - 12^2}$$

$$x = \sqrt{400 - 144}$$

$$x = \sqrt{256} = 16cm$$



Per-unit Quantities

Mathematics Review

- Ratios play an important part in estimating power system performance
- A ratio is defined as a relationship between two numbers as a fraction
- Generally, ratios are used when the relationship of two pairs of values is the same, and one of two similarly related values is known
- Ratios only provide exact answers in linear systems where the relationship between two variables in the system is the same regardless of the magnitude of the two variables

Question 4

Assume that the loss of a 1000 MW generating unit will typically result in a 0.2 Hz dip in system frequency. Estimate the frequency dip for the loss of an 800 MW generating unit.

Mathematics Review

$$\frac{0.2Hz}{1000MW} = \frac{(x)Hz}{800MW}$$

$$\frac{0.2Hz}{1000MW} \qquad = \frac{(x)Hz}{\cancel{800MW}}$$

$$\frac{(0.2Hz)(800\cancel{MW})}{1000\cancel{MW}} = (x)Hz$$

$$x = \frac{160Hz}{1000} = .16Hz$$

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Question 5

Assume that the loss of a 500 MW generating unit will typically result in a 0.3 Hz dip in system frequency. Estimate the frequency dip for the loss of an 300 MW generating unit.

Mathematics Review

$$\frac{0.3Hz}{500MW} = \frac{(x)Hz}{300MW}$$

$$\frac{0.3Hz}{500MW} = \frac{(x)Hz}{\cancel{300MW}}$$

$$\frac{(300\cancel{MW})(0.3Hz)}{500\cancel{MW}} = (x)Hz$$

$$x = \frac{90Hz}{500} = .18Hz$$

Mathematics Review

- Quantities on the power system are often specified as a percentage or a per-unit of their base or nominal value
- Per-unit values makes it easier to see where a system value is in respect to its base value and also how it compares between different parts of the system with different base values
- Per-unit values also allow for a dispatcher to view the system and quickly obtain a feel for the voltage profile

Mathematics Review

- Assume that, at a certain substation, the voltage being measured is 510 kV on the 500 kV system. What is its per-unit value with respect to the nominal voltage?

  Base or nominal voltage = 500 kV

  Measured voltage = 510 kV

  510 kV / 500 kV = <u>1.02 per-unit</u> or 102%

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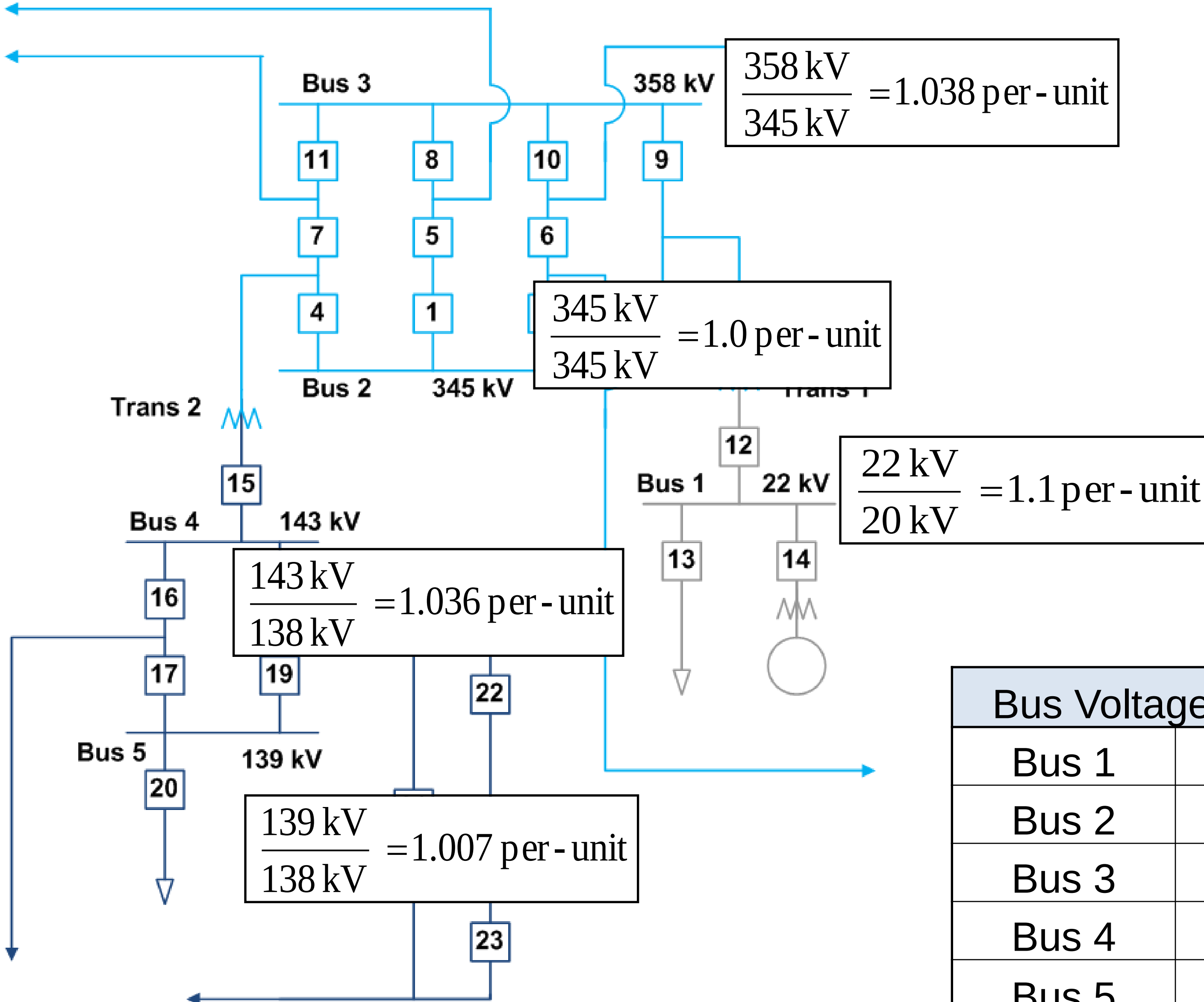
Mathematics Review

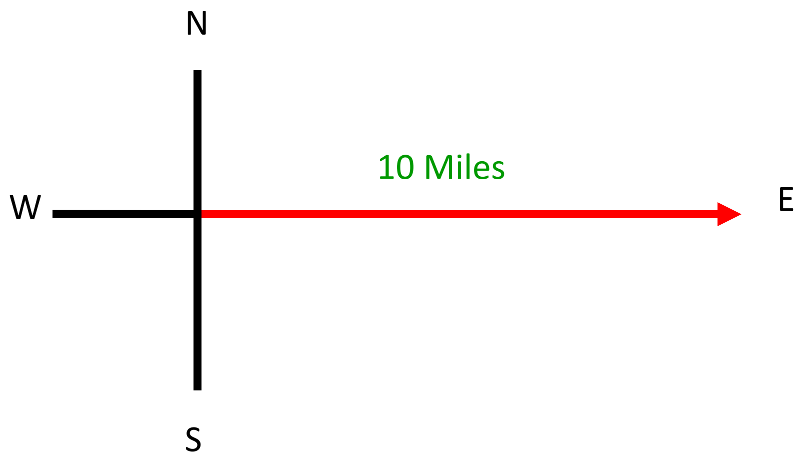


Bus Voltage Legend	
Bus 1	20kV
Bus 2	345kV
Bus 3	345kV
Bus 4	138kV
Bus 5	138kV

Vectors

Vectors

- A vector is alternative way to represent a sinusoidal function with amplitude, and phase information
- A vectors length represents magnitude
- A vectors direction represents the phase angle
- Example: 10 miles east



Vectors

- Vectors are a means of expressing both magnitude and direction
- Horizontal line to the right is positive; horizontal line to the left is negative
- Vertical line going up is positive; vertical line going down is negative
- Arrowhead on the end away from the point of origin indicates the direction of the vector and is called the displacement vector
- Vectors can go in any direction in space

Vectors

- The difference between a scalar quantity and a vector:

  a) A scalar quantity is one that can be described with a single number, including any units, giving its size or magnitude

  b) A vector quantity is one that deals inherently with both magnitude and direction

Conceptual Question 6

There are places where the temperature is $+20^o$ C at one time of the year and -20^o C at another time. Do the plus and minus signs that signify positive and negative temperatures imply that temperature is a vector quantity?

Question 7

Which of the following statements, if any, involves a vector?

a) I walked two miles along the beach

b) I walked two miles due north along the beach

c) A ball fell off a cliff and hit the water traveling at 17 miles per hour

d) A ball fell off a cliff and traveled straight down 200 feet

e) My bank account shows a negative balance of -25 dollars

Vectors

- When adding vectors, the process must take into account both the magnitude and direction of the vectors
- Vectors are usually written in bold with an arrow over the top of the letter, ($\vec{\mathbf{A}}$)
- When adding two vectors, there is always a resultant vector, R, and the addition is written as follows:

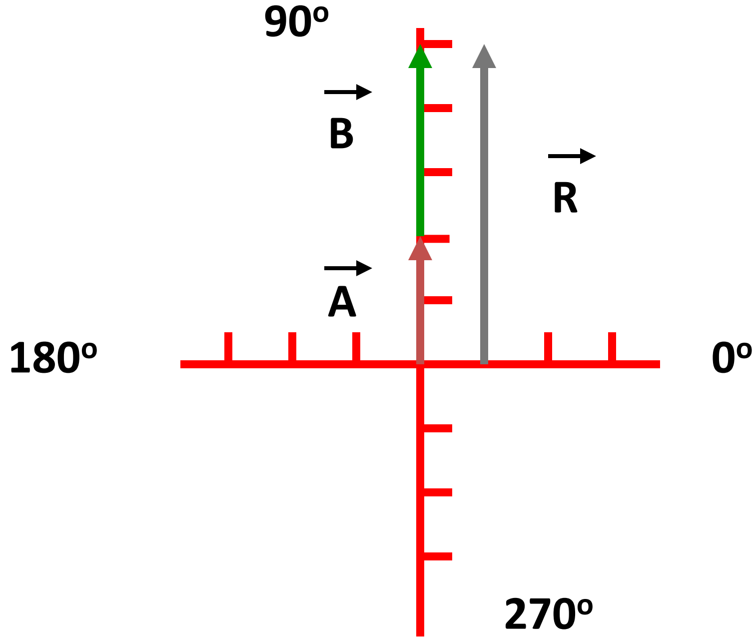
$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

Vectors

- Example: Adding vectors in the same direction

  Vector A has a length of 2 and a direction of 90°

  Vector B has a length of 3 and a direction of 90°

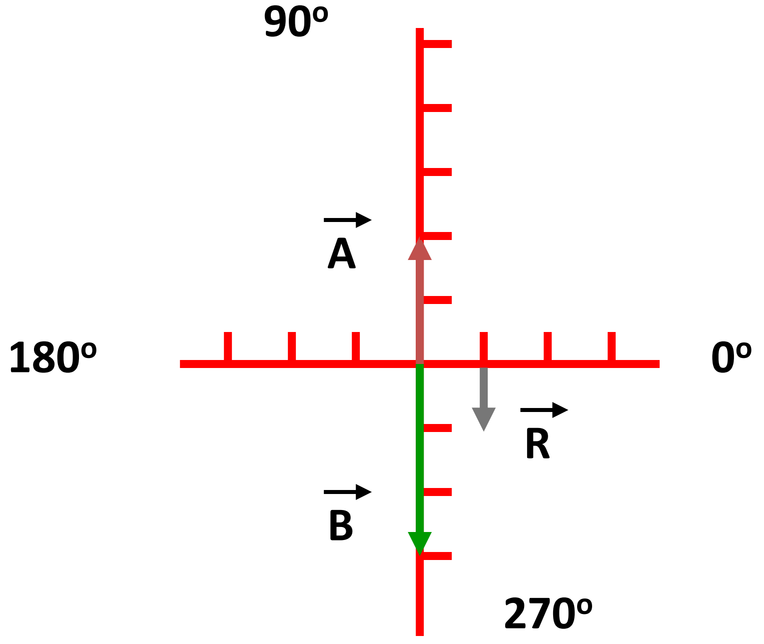


Vectors

- Example: Adding vectors in the opposite direction

  Vector $\vec{A}$ has a length of 2 and a direction of 90^o

  Vector $\vec{B}$ has a length of 3 and a direction of 270^0

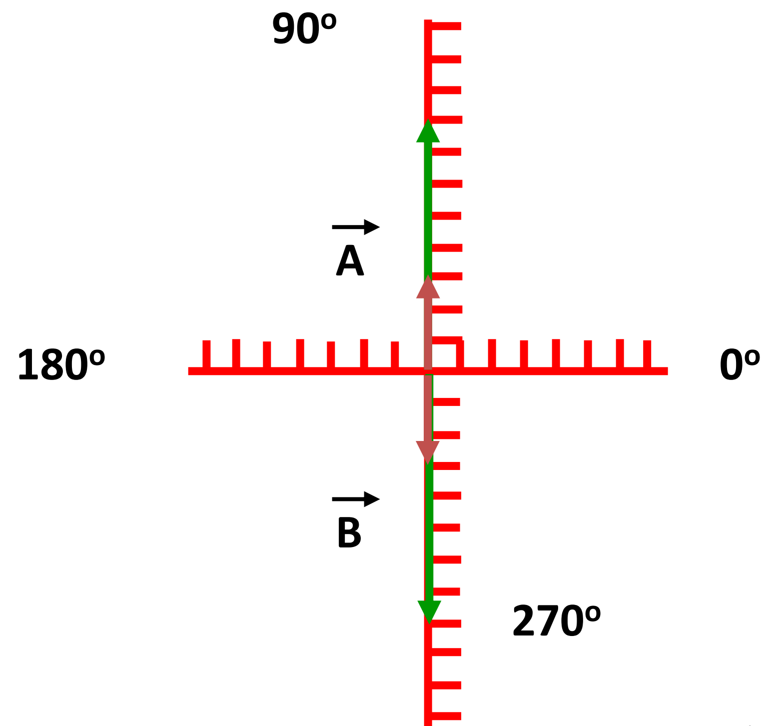


Question 8

Two vectors, $\vec{A}$ and $\vec{B}$, are added to give a resultant vector, $\vec{R}$. The magnitudes are 3 and 8 meters, respectively, but the vectors can have any orientation. What is the maximum and minimum possible values for the magnitude of $\vec{R}$?

Maximum: 3+8=11

Minimum: (-3)+(-8)=(-11)



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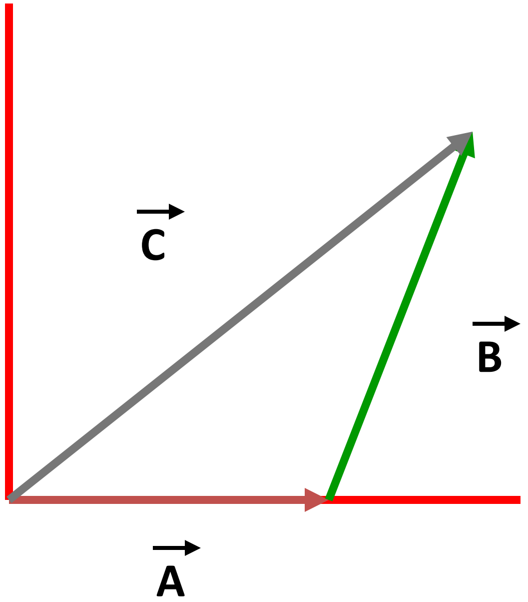
Vectors

- Subtraction of one vector from another is carried out in a way that depends on the following:

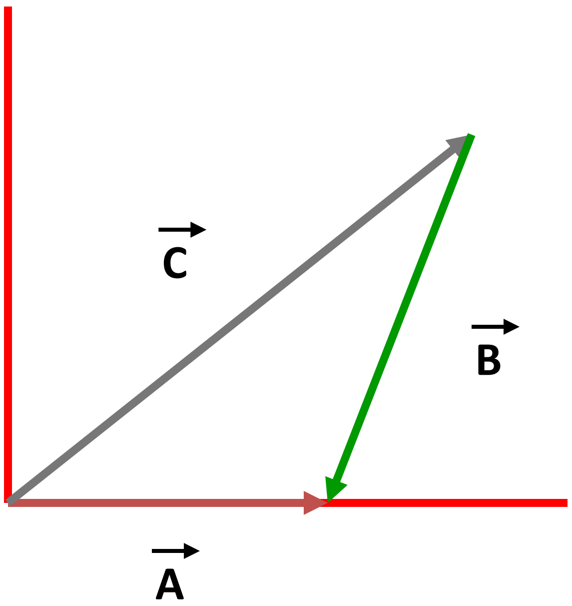
  When a vector is multiplied by -1, the magnitude of the vector remains the same, but the direction of the vector is reversed

- Vector subtraction is carried out exactly like vector addition except that one of the vectors added is multiplied by the scalar factor of -1

Vectors



$\vec{C} = \vec{A} + \vec{B}$



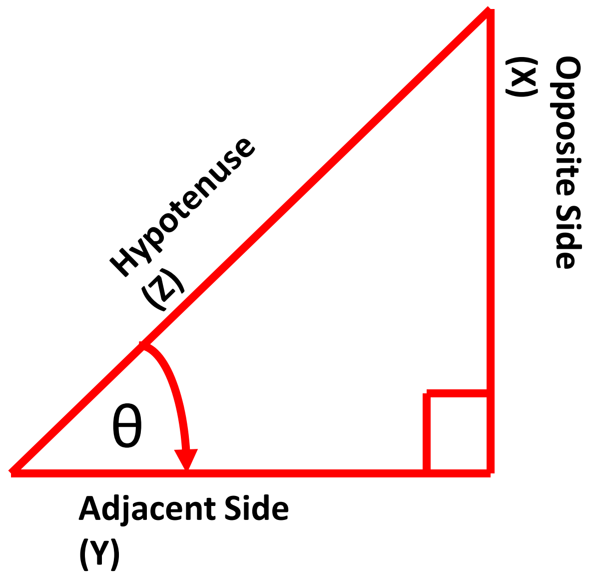
$\vec{A} = \vec{C} + (-\vec{B})$

Vectors

- If the magnitude and direction of a vector is known, it is possible to find the components of the vector
- The process is called "resolving the vector into its components"
- If the vector components are perpendicular and form a right triangle, the process can be carried out with the aid of trigonometry

Vectors

- To calculate the sum of two or more vectors using their components (x and y) in the vertical and horizontal directions, trigonometry is used



Vectors

- The Pythagorean Theorem is a special relationship that exists in any triangle and describes the relationship between the lengths of the sides of a right triangle

$$z^2 = x^2 + y^2$$

- Three basic trigonometric functions defined by a right triangle are:

$\sin\theta = x/z$ = opposite side/hypotenuse

$\cos\theta = y/z$ = adjacent side/hypotenuse

$\tan\theta = x/y$ = opposite side/adjacent side

$\tan\theta = \text{sine}\ \theta/\cos\theta$

Vectors

- To find theta, the inverse of the trigonometric function must be used

$$\theta = \sin^{-1} y/z$$

$$\theta = \cos^{-1} x/z$$

$$\theta = \tan^{-1} y/x$$

- When adding vectors, magnitude is found by:

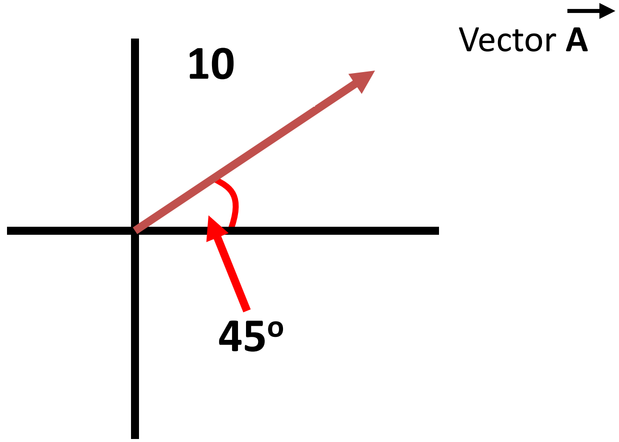
$$\vec{R} = \sqrt{(\vec{R_x})^2 + (\vec{R_y})^2}$$

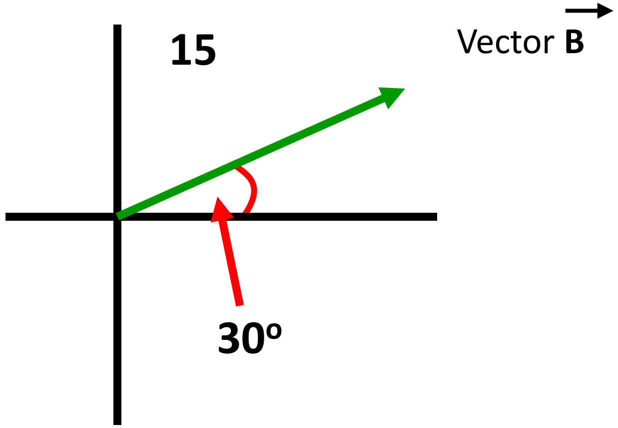
- The direction of the resultant, R, is found by:

$$\theta = \sin^{-1} (R_y/R)$$

Vectors

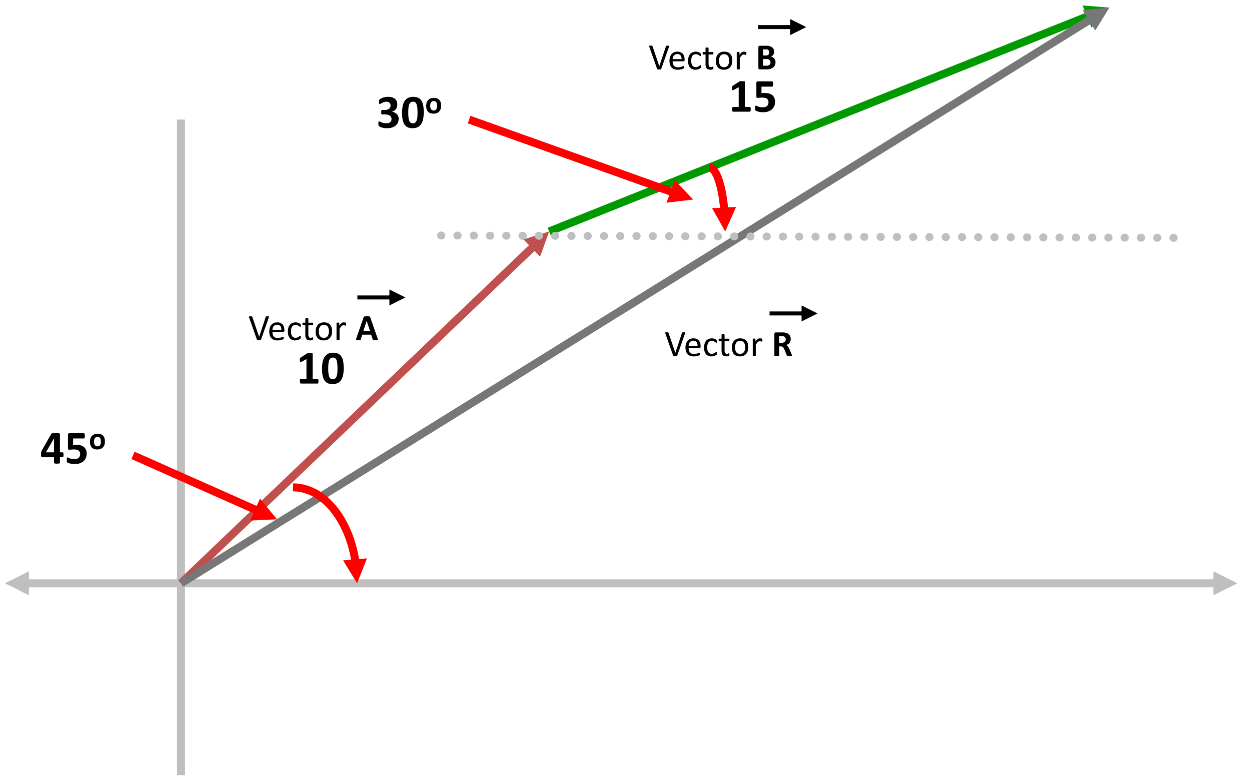
- Vectors can be added either in the same direction or in opposite directions



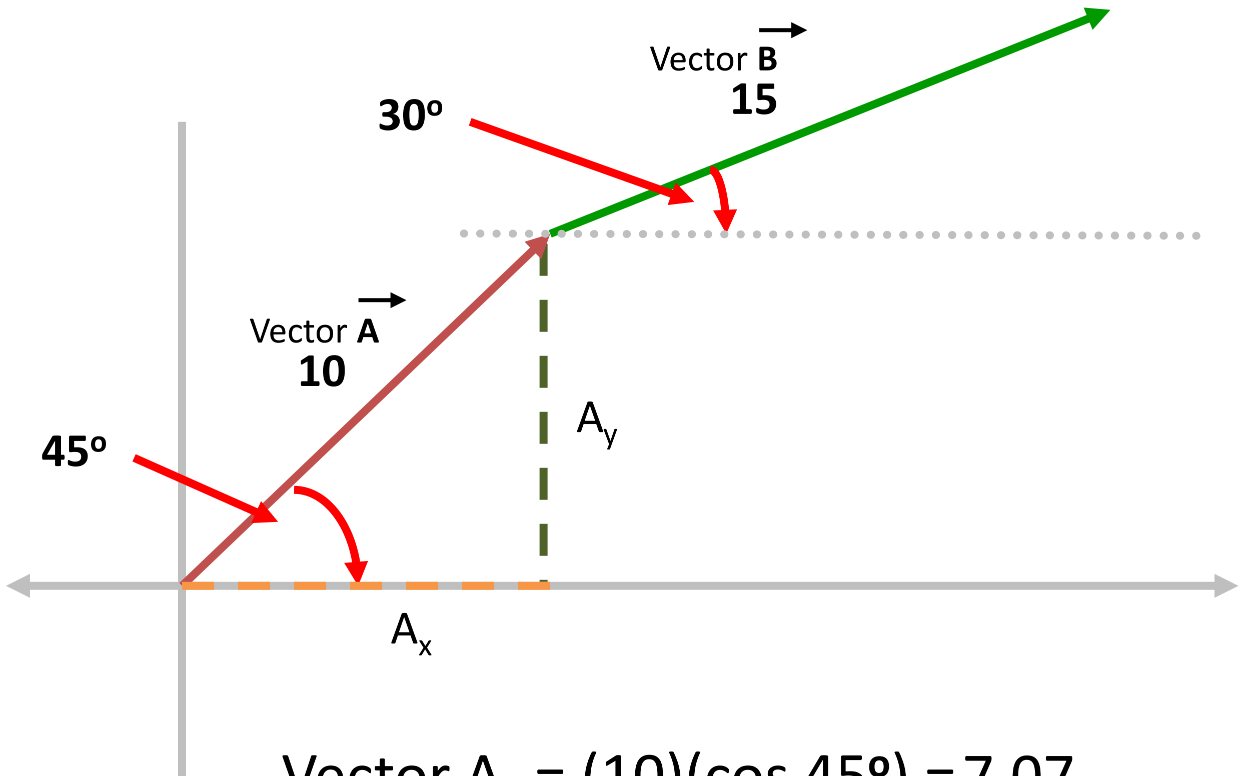


Vectors

- Adding vectors:

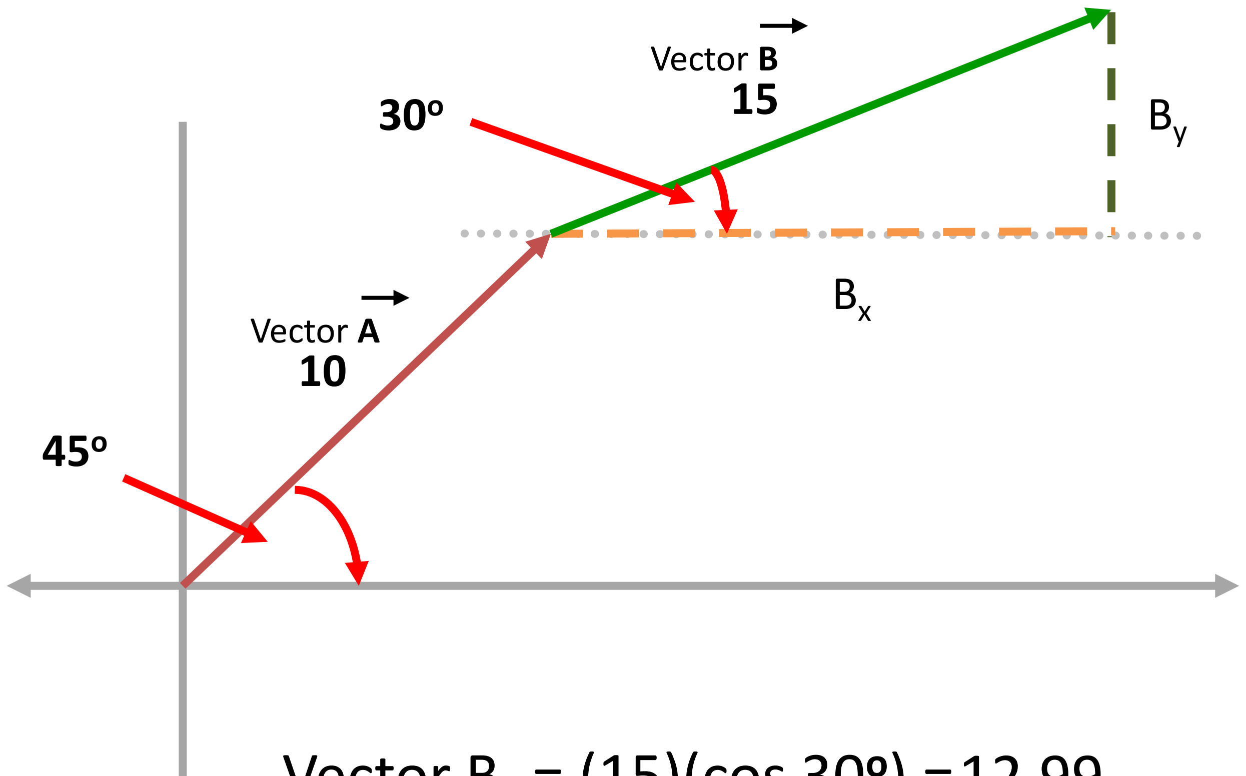


Vectors



Vector A_x = (10)(cos 45°) = 7.07
Vector A_y = (10)(sin 45°) = 7.07

Vectors



Vector B_x = (15)(cos 30º) =12.99
Vector B_y = (15)(sin 30º) =7.5

Vectors

- Determine the resulting vectors, R_x and R_y:

$\vec{R_x} = \vec{A_x} + \vec{B_x}$

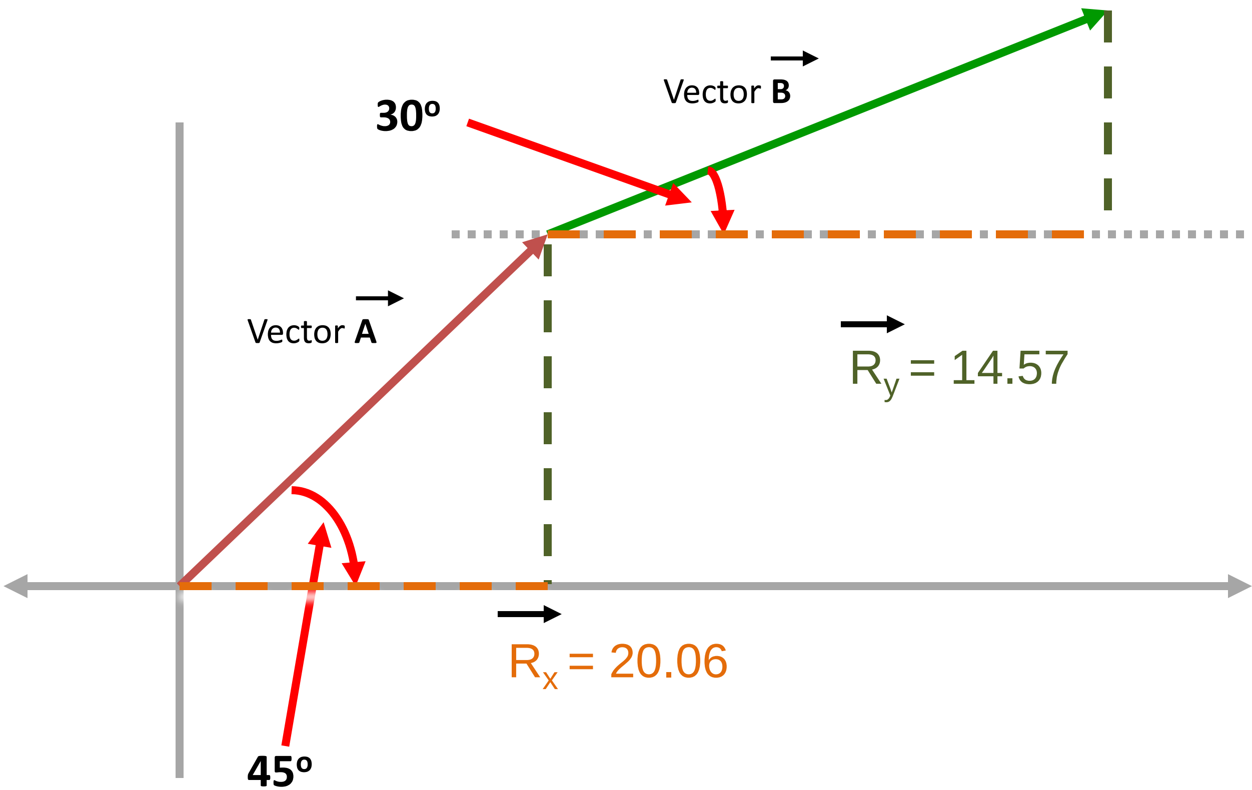
$\vec{R_x} = 7.07 + 12.99$

$\vec{R_x} = 20.06$

$\vec{R_y} = \vec{A_y} + \vec{B_y}$

$\vec{R_y} = 7.07 + 7.5$

$\vec{R_y} = 14.57$



Vectors

$$\vec{R} = \sqrt{\vec{R}_x^2 + \vec{R}_y^2}$$

$$\vec{R} = \sqrt{20.06^2 + 14.57^2}$$

$$\vec{R} = \sqrt{614.7}$$

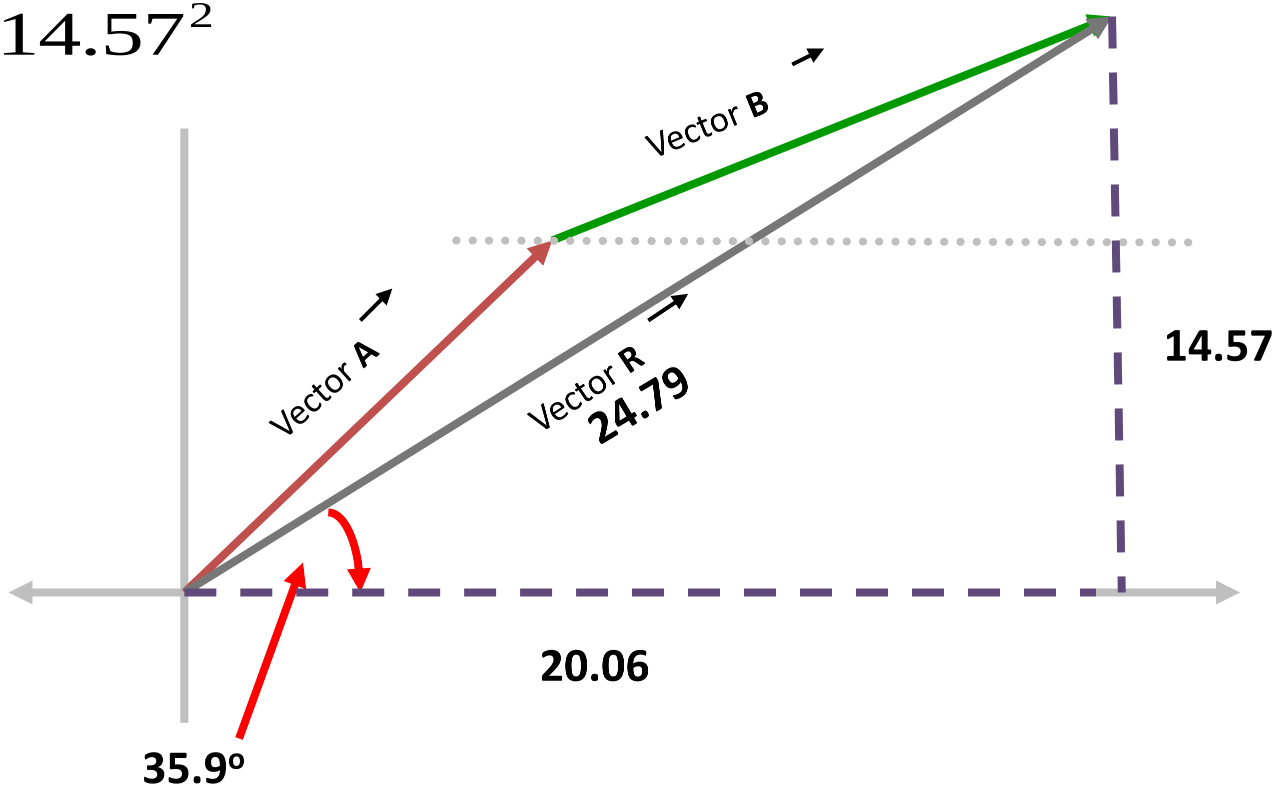
$$\vec{R} = 24.79$$

$$\theta = \sin^{-1}\left(\frac{R_y}{R}\right)$$

$$\theta = \sin^{-1}\left(\frac{14.57}{24.79}\right)$$

$$\theta = \sin^{-1}(.587)$$

$$\theta = 35.9°$$



Vectors

- Polar notation expresses a vector in terms of both a magnitude and a direction such as:

  where:

$$M \angle {}^{\circ}$$

M is the magnitude of the vector

^{o} is the direction in degrees

Example: Vector with a magnitude of 10 and a direction of -40 degrees

$$10 \angle -40^{o}$$

Vectors

- Multiplication in polar notation:

  Multiply the magnitudes/Add the angles

  $(50 \angle 25^o)(25 \angle 30^o) = 1250 \angle 55^o$

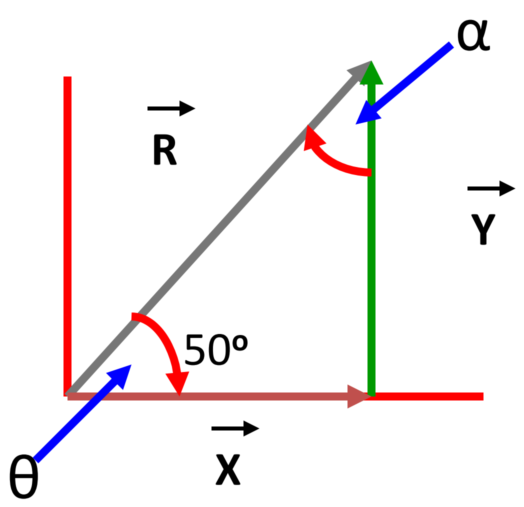
- Division in polar notation:

  Divide the magnitudes/Subtract the angles

  $$\frac{(50 \angle 25^o)}{(25 \angle 30^o)} = 2 \angle -5^o$$

Question 9

A displacement vector $\vec{R}$ has a magnitude of $\vec{R}$ = 175 m and points at an angle of 50° relative to the x-axis. Find the x and y components of this vector?

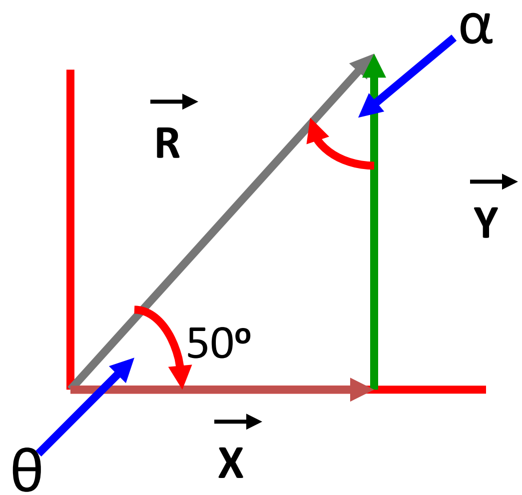


Question 9

$$\sin\theta = \frac{O}{H} = \frac{Y}{R}$$

$$\sin 50^\circ = \frac{Y}{175\ \text{m}}$$

$$Y = (175\ \text{m})\sin 50^\circ = 134.06\ \text{m}$$



$$X = \sqrt{H^2 - O^2} = \sqrt{R^2 - Y^2}$$

$$X = \sqrt{175\ \text{m}^2 - 134.06\ \text{m}^2} = \sqrt{12652.92\ \text{m}^2} = 112.5\ \text{m}$$

Summary

- Discussed the different components of a Right Triangles
- Reviewed the basics of Trigonometry
- Computed different Per-Unit Quantities
- Characterized the two components of Vectors

Questions?

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